STUDY OF BRIDGE VIBRATIONS FOR CONNECTICUT

Final Report

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PREFACE

This is the project report for "Study of Bridge Vibrations for Connecticut," Project 84-1, sponsored by the Joint Highway Research Advisory Council. John T. DeWolf and Edward V. Gant were the principal investigators, and Susan M. LaShomb and Jine-Wen Kou were graduate research assistants.

With the exception of Chapter 4 on the dynamic behavior, this report is based on the work presented in Susan LaShomb's thesis submitted to the University of Connecticut for her Master of Science in 1985. The literature review, reported in Chapter 2, is presented in considerably more detail in her thesis. Chapter 4 has been prepared by Jine-Wen Kou and represents the initial part of his work on the dynamic analysis of the Founders Bridge.

A subsequent proposal has been prepared to continue this study. This will involve the actual field monitoring and the development of a two dimensional dynamic analysis of the Founders Bridge. The study will include a determination of whether the vibrations are detrimental to the Founders Bridge and whether there are unique features of this bridge which are causing the objectionable vibrations. A general discussion of the influence of bridge vibrations on behavior and guidelines for evaluating bridges will be included in the final report for this study.

The assistance of the Connecticut Department of Transportation in this study is gratefully acknowledged.

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Chapter 1

INTRODUCTION

1.1 PURPOSE AND SCOPE

The State of Connecticut has the responsibility for inspecting and maintaining approximately 4800 bridges. The service life of these structures is dependent on a number of circumstances, including the type of structure, the effects of the environment on the bridge and the load history. It has been the purpose of this study to investigate the effect of vibrations from traffic loading, i. e. vertical accelerations, on the service life of bridges.

Vibrations have become an increasingly important factor in the design of bridges, in part due to the development of new, higher strength materials which result in lighter structures, in part due to refined design and analysis procedures which allow the use of increasingly slender members, and in part due to increasing vehicle loads. The research on vibrations has not kept up with these factors, and what has been done is not compiled in a readily available form for use by engineers to evaluate the actual performance of highway bridges.

While the vibrations do not often result in collapse, they can be a factor in fatigue failures. They can also contribute to concrete deck problems. Vertical accelerations have been shown to be significant in producing undesirable psychological effects on pedestrians and occupants of stopped vehicles.

The Connecticut Department of Transportation receives reports from the public that certain bridges feel unsafe due to vibrations. The Department needs a procedure for evaluating such reports. An example is the Founders

Bridge across the Connecticut River at Hartford. This is a continuous span bridge with two main steel support girders positioned longitudinally under the center of the three lanes in each direction perpendicular to the highway. The continuity and the cantilevered portions complicate the dynamic behavior and make simple predictions impossible.

The scope of this study has involved the assemblage of information on bridge vibrations due to vehicles and an initial study of the Founders Bridge and its susceptibility to vibrations. The goal was to develop the information to write a further proposal for a full scale field study. The limits in the analytical work reported in the literature along with the lack of a suitable analytical technique for the study of a bridge with the complexity of the Founders Bridge has demonstrated the need for the development of a software package for the two dimensional analysis of bridge vibrations.

1.2 STUDY OBJECTIVES

The first objective of this study was to review the literature available on bridge vibrations. This included: (1) reports of studies on bridges, including both field studies and analytical studies; (2) the collection of all proposed guidelines on the determination of the effects of vibrations on the structural response, including code provisions used in the design of bridges; (3) a review of monitoring systems used in field studies of bridges.

The literature study was used to define the parameters that affect bridge vibrations due to gravity loading. These can be broken into the following categories: (1) bridge parameters, such as material characteristics, beam span and stiffness; (2) vehicle parameters, such as velocity and transverse position of the wheels; (3) construction parameters, such as roadway roughness. The number and complexity of these parameters has demonstrated that present design practice, with vibration treated by limiting the static deflections and the span/depth ratio, is overly simplistic. It has been shown that these design provisions do not necessarily ensure the comfort of bridge users nor do they ensure that structural problems will not occur.

The second objective of this project was the study of the Founders Bridge. This was to include an initial analytical study and the proposal of a monitoring system to conduct a field study.

The analytical study involved the preparation of the data for stiffness analysis of the bridge and preliminary estimates of the static deflections, needed to define limits for the subsequent dynamic field study.

In addition, natural frequencies and mode shapes were determined. This involved both a one-dimensioal analysis and the attempt at a two-dimensional analysis. The extensive review of the literature demonstrated the lack of suitable software for two-dimensional vibrational studies and the severe limits on the ability to apply moving loads to bridge analyses.

In addition to the analytical study, a goal was the development of a monitoring system for the actual field study of the Founders Bridge. This was based on the review of similar studies reported in the literature and a study of available monitoring systems. The monitoring system should quantify the vibrations and provide this data to determine if design limits on stresses or deflections are exceeded.

1.3 OUTLINE OF THE REPORT

The literature review is summarized in Chapter 2. Included are reports on field studies of steel bridges, similar to the Founders Bridge, and other types of bridges. Also included is a review of model studies and a review of analytical work. Chapter 3 contains a summary of design recommendations and proposed guidelines contained in codes and based on conclusions from previous research studies.

The results of a free vibrational study are contained in Chapter 4. This involved separate one-dimensional studies of the bridge in the longitititudinal and transverse directions.

Finally, a brief summary of this work is given in Chapter 5. This includes both the major parameters that influence bridge vibrations, as determined from the literature search, and the general approach needed for a field study of the Founders Bridge.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Bridge vibration has been a concern to bridge designers and users for years. Much of the earlier studies of bridge vibration were begun after bridge failures due to vibration. Perhaps the most well known was Tacoma Narrows Bridge failure due to lateral vibrations from wind. Many of the problems however have only recently been addressed, through analytical and field studies.

Analytical equations to describe bridge vibration behavior were initially developed by C. E. Inglis for railroad bridges and S. Timenshenko for suspension bridges in the early 1900`s. In the 1950`s and early 1960`s actual bridge tests, particularly of simple single span bridges, were undertaken to study bridge vibrations and to understand the parameters involved. Also during this time, model testing in the laboratory and simplified beam and moving force analytical models were used to study bridge vibrations. As computers became more available in the 1960`s and 1970`s, much of the analytical work was done in developing computer models to describe both bridge vibrations and vehicles with sprung and unsprung mass models. During the same period, actual field tests of bridges began on more complicated bridges such as continuous bridges.

Currently, bridge vibration studies include: suspension bridges, cable-stayed bridges, curved bridges, prestressed concrete bridges, and high speed transit structure bridges. Analytical studies and computer models are being used to handle such items as acceleration and deceleration of vehicles, surface roughness of the bridge deck, and multi-vehicle loading.

Field studies are covered in Sections 2.2 and 2.3 for steel girder bridges and other types of bridges, respectively. Section 2.4 contains the review of model studies, and this is followed by a discussion of analytical papers in Section 2.5. Probability studies, developed to determine random approaches for modeling the loads, are reviewed in Section 2.6. A summary based on the literature review, is then given in Section 2.7.

2.2 FIELD TESTING: STEEL HIGHWAY BRIDGES

Much of the initial work in bridge vibration began with actual bridge testing. This enabled the researchers to identify the factors that most affected bridge vibrations. The initial tests involved simple span bridges only. This was followed by the study of multispan bridges and then by more complicated bridges, such as suspension bridges.

There were five simple span dynamic bridge field studies. K. H. Kinnier and W. T. McKeel (53,60) looked at the influence of the bridge substructure on the dynamic behavior. Their first study compared conventional and elastomeric influence bearings on a rolled-beam, composite bridge. In their second study, they compared tall and short piers on a composite bridge by measuring strains and deflections caused by a 3-axle tractor-trailer. The earliest of the other three studies involved AASHO Road Tests (96). These involved 18 three beam bridges with 14 different vehicles. Both deflections and strains were measured. The next study was by J. M. Biggs and H. S. Suer (10). They tested two steel stringer bridges and three plate-girder bridges with the load applied by a 2-axle, 10 ton dump truck. They measured deflections with a deflectometer and accelerometers. The remaining study was by J. M. Biggs (9). He tested two single medium span stringer type through girder bridges, with the load applied by a 2-axle dump truck. The deflection was measured by deflectometers at midspan, and strains and accelerations were measured on the axles of the truck.

Seven field studies were made involving both simple and continuous span bridges. J. R. Billing and R. Green (11) reviewed three series of dynamic tests done in Ontario, Canada in 1956 to 1957, 1969 to 1971, and 1980. In the first two series, deflection was measured, while in the last series, acceleration was measured. In the last two series, test vehicles were used. C. P. Heins (49) tested 40 bridges with five different types of vehicles obtaining stress versus time records at several locations along the bridge. R. Green (41) summarized 52 bridge tests involving with both a test vehicle and normal traffic loading. He measured the deflections with a deflectograph. Eyre and Tilly (32) tested 23 bridges for damping characteristics by exciting 22 of them with reciprocating weights and one by dropped weights. C. G. Schilling, K. H. Klippstein, J. M. Barsom, and G. T. Blake (81) studied fatigue in bridges by summarizing 15 field bridge tests subject to traffic loading. Perhaps the most complete study was that by J. T. Gaunt and C. D. Sutton (37). They tested 62 bridges under normal traffic and a test vehicle. Both accelerations and deflections were measured.

Ten field studies were made specifically on continuous span bridges. R. C. Edgerton and G. W. Beecroft (29,30) studied two 3-span plate girder bridges with a tractor trailer combination. They measured strains for a variety of loading cases. At about the same time as Edgerton and Beecroft`s study, J. M. Hayes and J. A. Sbarounis (48) tested a 3-span, continuous, I-beam bridge by measuring strains due to vibrations caused by a test truck. G. M. Foster and L. T. Oehler (34) tested an 8-span plate girder bridge and a 6-span rolled beam bridge with 2-axle and 3-axle trucks and normal traffic. They measured deflections with a deflectometer. The next study was by D. A. Linger and C. L. Hulsbos (59) who tested four continuous span bridges. The loading was a van truck and a tractor trailer combination, and response was measured by strain gages. H. S. Ward (103) studied a 5-span continuous, variable sized plate girder bridge with forced oscillation and a test vehicle. The vertical vibrations were measured with seisometers, i.e. velocity transducers. A study of strain measurements in an 11-span composite bridge, subject to a 40 ton truck was done by J. B. Menzies (64). In an extensive study W. H. Walker and J. S. Ruhl (102) tested two three-span continuous bridges and one two-span continuous bridge with normal traffic and a three axle tractor-semitrailer combo, with strain and deflection gage measurements. At about the same time, R. Eyre and I. J. Smith (31) used deflection gages to study a two level, 20-span steel box girder bridge with normal traffic and a test truck vehicle. R. S. Shepherd, H. E. E. Brown, and J. H. Wood (82), studied a 3-span steel truss bridge subject to wind, a dynamic exciter, and normal traffic loading. Measurements were made with dynamic displacement transducers and seisometers.

2.3 FIELD TESTING: OTHER BRIDGE TYPES

Vertical vibration studies were made on suspension bridges. All tested the bridges under ambient loading and all but the last study by Buckland, Hooley, Morgenstern, Rainer, and Van Selst (who used accelerometers), used seisometers to measure vibrations. V. R. McLamore (61) tested the Chesapeake Bay Bridge with the seisometers placed in five different patterns, with one seisometer always used as reference. V. R. McLamore, I. R. Stubbs, and G. C. Hart (62) tested the Newport Rhode Island Suspension Bridge in the same way. A. M. Abdel-Ghaffar and G. W. Housner (2,3) tested the Vincent-Thomas Suspension Bridge in California. They used 16 different set ups of the eight seisometers with 4 of the seisometers always placed in the reference locations. P. G. Buckland, R. Hooley, B. D. Mogenstern, J. H. Rainer and A. M. Van Selst (15) used the Fourier spectra from their data to calculate damping, natural frequencies and mode shapes.

There were six field studies on concrete bridges. The first study was by J. Tibor (88) who studied 3 and 5 span prestressed concrete bridges with two

20 ton test trucks driven at varying speeds. The response was measured by deflectometers at the quarter and half points of each span. R. F. Varney (99) studied a 2 span prestressed concrete bridge by measuring strains, deflections, and accelerations when a tractor trailer crossed at varying speeds. A test on a 3-span concrete bridge, with traffic and forced vibration loading by J. M. Rainer and G. Pernica (74), measured vibrations with seisometers. P. J. Moss, A. J. Carr, and G. C. Pardoen (68,69) studied three prestressed concrete I-beam bridges by accelerometers and seisometers during normal traffic, forced vertical loading produced by people jumping, and forced lateral loadings. R. Cantieni (16,17) summarized 23 years of bridge testing by the Swiss.

Studies were also done on a railroad bridge, transit structures, pedestrian bridges, and pavement loads. V. Kolousek (54) tested a 3-span railroad truss bridge with a locomotive and measured strains deflections. The pavement load study was done by A. P. Whittemore, J. R. Wiley, P. C. Schultz, and D. E. Pollock (108). They used both electronic scales embedded in the pavements and on board measurement systems to measure dynamic behavior. M. L. Silver and T. Venema (83) tested three elevated transit structures with accelerometers and three types of transit vehicles as loading. J. E. Wheeler (107) studied 21 pedestrian bridges using people jumping and people walking as the loading.

2.4 MODEL TESTING

Model bridge studies were performed in the laboratories to determine the relationships between different parameters. Many of these model studies were done in conjunction with field or analytical studies so that the results could be compared. Model testing has been particularly effective since it simplifies the actual structure so that important parameters can be isolated and studied more thoroughly.

Four studies were made of simple span bridge models. The first was by T. P. Tung, L. E. Goodman, T. Y. Chen, and N. M. Newmark (95) who tested a beam with strain gages every four inches being crossed by a load carriage. M. Naruoka and H. Y. Yoneza (72) measured free transverse vibrations and periods of a beam with a cathode-ray oscilloscope in their model study. A bridge model of a weighted steel beam plus an aluminum chassis model vehicle was set up by J. M. Biggs (9) to measure strains. K. T. S. R. Iyengar and R. N. Iyengar (51) studied five aluminum model beam and slab bridges excited with an electromagnetic device, measuring strains.

Four studies were made on continuous span bridge models. R. S. Ayre, G. Ford, and L. S. Jacobsen (6) studied a two span beam and carriage and measured strains. J. F. Fleming and J. P. Romualdi (33) tested a three span continuous beam with a moving load at selected velocities. Beam and joint combinations were tested for damping by G. P. Tilly (89). J. F. Wilson (110) and T. P. Joseph (52), used a weighted steel beam and a linear induction powered model vehicle with accelerometers fore and aft.

F. Van der Woude (97,98) made a model of a suspension bridge to measure deflections and strains.

2.5 ANALYTICAL STUDIES

Unlike field and model studies, analytical studies have included many different methods and have evolved to more sophisticated treatments. Initially, analytical methods started with a simple differential equation, based on a one-dimensional model that was solved by major assumptions with energy methods. The analytical methods have progressed to include a wide range of variables, complex modeling, and solutions by finite element analysis or modal superposition on a computer. Only limited work has been done with two-dimensional analyses, however.

For the analytical studies, simple span and continuous span bridges were initially covered. The studies have included concrete bridges, suspension and cable-stayed bridges, transit structures, curved bridges, railroad bridges, and cantilever bridges.

There were 18 analytical studies of simple span bridges. Three of the studies, M. Narouka and H. Yonezawa (72), K. T. S. R. Iyengar and R. N. Iyengar (51), and by C. N. Kostem (56), determined free vibrations of bridges by modeling them as plates and assuming the deflection was a trigonometric function. R. K. Gupta and R. W. Traill-Nash (43) also modeled the bridge as an orthotropic plate, but they included vehicle effects. K. H. Lie (58) modeled a truss bridge as a continuous space structure and found the free vibrations using the finite element method. All the other studies modeled the bridge as a beam. Studies by R. K. Wen (104), by R. K. Wen and A. S. Veletsos (106), by T. F. Derby and P. C. Calcaterra (25), and by T. E. Blejevas, C. C. Feng, and R. S. Ayre (13) included surface roughness in their models. A. S. Dmitriev (26) modeled the vehicle in terms of the moment produced and solved for vibrations by assuming that the deflection was a trigonometric function. Studies by C. F. Scheffey (79), T. P. Goodman, T. Y. Chen, and N. M. Newmark (95), E. C. Ting, J. Genin, and J. H. Ginsberg (91), and T. E. Blejevas, C. C. Feng, and R. S. Ayre (13). involved a mass moving at constant velocity. S. I. Suzuki (85) modeled an accelerating vehicle as a mass moving at a constant acceleration, while R. K. Gupta (42) studied braking vehicles with a planar two axle sprung mass system model. Many studies modeled the vehicle as a combination of sprung and unsprung masses with springs and dampers. These were by T. P. Tung, L. E. Goodman, T. Y. Chen, and N. M. Newmark (95), J. M. Biggs (9), R. K. Wen (104), AASHO Road Tests (96), R. K. Wen and A. S. Veletsos (106), T. F. Derby and P. Calcaterra (25), and T. E. Blejevas, C. C. Fen, and R. S. Ayre (13). W. Zuk (115) studied natural vibrations of a beam and bearing system and E. Kosko (55) studied natural vibrations that were coupled flexurally and torsionally in a beam.

For continuous bridges, there were seven analytical vibration studies. Studies by A. C. Eberhardt and W. H. Walker (28), T. A. Rock and E. Hinton (78), and R. Shepherd, H. E. E. Brown, and J. H. Wood (82) modeled the bridge as a series of plate elements. Three of the studies modeled the bridge as a beam. A study by H. Hawk and A. Ghali (45) used a grid system with a lumped mass matrix to model the bridge. H. S. Ward (103), T. A. Rock and E. Hinton (78), and R. Shepherd, H. E. E. Brown, and J. H. Wood (82) were interested in the free vibrations. In studies by R. S. Ayre, George Ford, and L. S. Jacobsen (6), and D. A. Linger and C. L. Hulsbos (59), the vehicle was modeled as a massless moving force. Both studies by A. C. Eberhardt and W. H. Walker (28) and H. Hawk and A. Ghali (45) modeled the vehicle as a combination of sprung and unsprung masses.

There were seven studies on bridges with single and multiple spans. Two of the studies, by J. T. Gaunt, T. Aramraks, and M. J. Gutzwiller and R. H. Lee (36), and by J. T. Gaunt and C. D. Sutton (37), modeled the simple span bridges as a plate continuous over flexible beams. In another, by R. Wright and W. Walker (113), the bridge was modeled as a grid system. In the other studies, the bridge was modeled as a beam with either point masses or uniform section properties. A study by R. S. Ayre, L. Jacoben, and C. S. Hsu (7) modeled the vehicle as a moving mass load while a study by J. Wills (109) considered only free and forced vibrations. The studies by J. Fleming and J. Romualdi (33), A. Veletsos and T. Huand (100), J. T. Gaunt, T. Aramraks, and M. J. Gutzwiller (36); and J. T. Gaunt and C. D. Sutton (37) modeled the vehicle as a combination of sprung and unsprung masses with springs and dampers.

Only limited studies have involved concrete bridges. R. F. Varney (99) studied analytically and conducted a field study of a two span partially continuous reinforced bridge. P. J. Moss and A. J. Carr (68) conducted a similar study of a five span structure. Three prestressed simple spans were studied analytically by P. J. Moss, A. J. Carr and G. C. Pardoen (69).

There were eight analytical studies on vertical bridge vibrations in suspension bridges and on cable-stayed bridges. N. F. Morris (65,66,67) studied lumped mass two dimensional and three dimensional bridge models to find the free vibrations, while M. S. Causevic (18) studied a discrete mass system bridge model to find free and forced natural vibrations. D. B. Steinman (84) derived a simplified equation to calculate simple and superimposed harmonic oscillations of suspension bridges. Three of the studies on suspension bridges used energy principals to derive the natural free vibrations. These were by H. Reissner (76), F. Van der Woude (97,98), and A. M. Abdel-Ghaffar (1,2,3). In studies by J. Vellozi (101) and T. Hayashikawa, N. Watanabe, K. Sato, and H. Oshima (46,47), suspension bridge vibrations were defined using the deflection theory under free vibrations, constant moving load, and pulsating moving load.

Seven analytical studies were made for transit structures. E. C. Ting and M. Yener (92) present a summary of transit structure vibration studies. The rest of the studies modeled the transit structure as a beam and used the Bernoulli-Euler beam theory in their derivation. T. P. Joseph and J. Wilson (52) studied curved transit structures under single and tandem point loads. Studies by H. H. Richardson and D. N. Wormely (77) and by J. Genin and Y. I. Chung (38), included surface irregularities of the beam in their derivation. James F. Wilson (111) modeled the vehicle as an unsprung weight moving at a constant velocity. In other studies, by H. H. Richardon and D. N. Wormely (77), by J. Genin, J. H. Ginsberg, and E. C. Ting (39), by the Committees of Structural Steel Producers and of Plate Producers (22), and by J. Genin and Y. I. Chung (38), the vehicle was modeled as sprung and unsprung weights on a spring.

There were eight curved bridge analytical studies. H. Yonezawa (114), used an orthotropic plate bridge model and Y. K. Cheung and M. S. Cheung (19), used a finite plate strip bridge model to find the free vibrations of curved bridges. The only study on free vibrations of skewed bridges was by C. N.

Kostem (57). The rest of the studies modeled the curved bridge as a curved beam and found free vibrations. In addition, in studies by C. P. Tan and S. Shore (86,87), by C. G. Culver (23) and P. P. Christiano (20), and by J. Genin, E. C. Ting, and Z. Vafa (40), the vehicle was modeled as a massless force and as sprung and unsprung masses connected with springs. To derive the equations, C. G. Culver and P. P. Christiano (20) used curved beam theory, while C. P. Heins and M. A. Sahin (50) used finite differences and Y. K. Cheung and M. S. Cheung (19) used the finite strip method.

Limited analytical studies have been made on railroad bridges. Kolousek (54) only modeled the engine as a constant moving load, while Chu, Garg, and Dhar (21) modeled the bridge as a three degree of freedom, four axle system. Kolousek used the slope-deflection theory to develop his equation of motion, while Chu, Garg, and Dhar used a lumped mass matrix development. Kolousek compared his work with actual field studies and found it inadequate, and Chu, Garg, and Dhar only conducted a numerical study.

Two analytical studies were made on cantilever bridges. Wen and Toridis (105) considered the bridge to be a lumped mass beam, while N. Nagaraju, K. S. Jagadish, and K. T. Iyengar (71) considered the bridge to be a beam with mass uniformly distributed along its length. Both studies considered a constant moving mass load, but Nagaraju, Jagadish, and Iyengar's study also treated a moving sprung load. Both studies found that cantilever bridges are the most susceptible type of bridge to vibrations. They found that vibrations could be minimized, if the suspended span was relatively large and the cantilever span was relatively small.

2.6 PROBABILISTIC STUDIES FOR LOADS

Two studies, by L. Fryba (35) and Tung (93,94), were concerned with loads in terms of probability functions for bridges while the one study, by A. P. Whittemore, J. R. Wiley, P. C. Shultz, and D. E. Pollock (108) dealt with pavement loads. The pavement load study discussed three different probability loading models and ran a parameter study. Fryba considered the total vehicle load as random. He modeled the loading as a random process. On the other hand, Tung considered all the vehicles to be of the same weight and speed and the number of vehicles entering the bridge to be the random process. The second modeling technique is more accurate. In a related paper, D. R. Schelling (80), looked at error bounds for analytical solutions.

2.7 SUMMARY AND CONCLUSIONS

1. Nine different types of bridges have been investigated:
    a. Simple Span Bridges - the most extensively studied type of bridge.
    b. Continuous Span Bridges.
    c. Suspension Bridges - field tested for normal traffic and weather conditions and analytically studied only for natural modes and frequencies.
    d. Cable-Stayed Bridges - analytical studies only for natural modes and frequencies.
    e. Cantilever Bridges - the type of bridge most susceptible to vibrations.
    f. Curved Bridges - only studied analytically, typically using curved beam theory.
    g. Railroad Bridges - few studies have been done recently on these bridges.
    h. Transit Structures - typically they are elevated with transit vehicles that travel faster than typical highway traffic, so that these types of structures are especially prone to vibrations.
    i. Pedestrian Bridges - since they are lighter than highway bridges, they are more prone to vibrations.
2. The bridges studied were made of a variety of materials with varying span lengths, number of spans, and construction techniques. Steel bridges were made with plate girders, rolled beams, box girders and trusses. Both composite and

noncomposite structures were studied, with 1 to 20 spans. Concrete bridge types included reinforced, prestressed, twin box structures, and I-beam. Both composite and noncomposite structures were studied, with 1 to 5 spans. The only timber bridges studied were those with simple spans. Limited work has been done with composite pedestrian bridges made with aluminum.

3. The types of loading applied in field tests were:
   a. Forced Lateral - to find lateral modes and frequencies of vibration; load applied by an electrohydraulic inertial shaker.
   b. Forced Vertical - to find vertical and torsional modes and frequencies of vibration (depending on the placement of the vibration inducing mechanism); the loads were applied by mechanical oscillators, reciprocating weights, and dropped weights.
   c. Natural Vibrations - to find the natural modes and frequencies; in field tests, the bridge was tested under ambient conditions (normal traffic and wind loading) by measuring vibrations with seisometers and running a Fast Fourier Transform Analysis.
   d. Normal Traffic - typically measured for specific time periods, with a variety of measurement locations.
   e. Light Winds - to find lateral modes of vibration.
   f. Pedestrian Loads - vibrations applied by: walking at normal and at specific frequencies, jogging, and jumping.
   g. Test Vehicle - to find the modes and frequencies; the vibrations were applied by: vehicles with 2 to 8 axles, vehicles with weights ranging from 21000 pounds to 75000 pounds, dump trucks, tractor-trailers (many were loaded as the H20-44 loading truck in AASHTO Specifications), school buses, vans, locomotives, and rapid transit vehicle.
4. For field tests, the following types of equipment were used:
   a. Vehicle Measurements:
      1) Spring Deflection Gage - measured by linear potentiometers.

2) Tire Pressure Gage - to estimate differences in the weight on each tire.

3) Accelerometers - to measure vehicle and axle weight.

b. Bridge Measurements:

1) The deflection was measured by dial gages, deflection gages, strain gages, deflectometers (cantilever bar with strain gages, fixed to a support off of the bridge), linear differential transformers, and dynamic displacement transducers (measured the amplitude of vibration).

2) The velocity was measured with seisometers.

3) The acceleration was measured with seisometers.

4) The relative slip was measured with dial gages to find the degree of composite action between the steel beams and the slab.

c. Recording Equipment:

1) Oscillograph Equipment - to record the response.

2) Amplifiers - to amplify the electronic response.

3) Signal Conditioners - to condition the response and keep spurious data out of recorded response.

4) Tape Recorders - to record the response.

5. Field records were processed for data reduction as follows: First, the field records on the oscillograph or the tape recorder were filtered and digitized so that they could be handled on a computer. Next, the curves were smoothed to remove spurious data. From the smoothed digitized record, peak values of deflection, velocity, acceleration, jerk (change in acceleration), and damping were found by integrating or differentiating the record. Finally, a Fourier Series Analysis was run to find mode shapes and frequencies.

6. For the analytical studies, the loading types included:

a. Natural Vibrations - lateral, torsional, and vertical.

b. Massless force moving at a constant velocity.

c. Smooth rolling mass moving at a constant velocity.

d. Mass moving at a constant acceleration.

e. One axle vehicle - combination of sprung load (connected to a spring and allowed to oscillate) and unsprung mass, with a spring between them, moving at a constant velocity.

f. Two and three axle vehicles - with and without damping, with both sprung loads and unsprung loads interconnected by springs, and vehicles moving at a constant velocity or braking.

7. The following lists the variables and summarizes the test behavior:

   a. Vehicle Factors:

      1) Axle spacing - when the time interval between 2 axles passing over a given point was equal to the natural period of the bridge, the dynamic effects increased.

      2) Springs blocked - doubles the impact factor over vehicles with their springs unblocked.

      3) Multiple vehicles crossing the bridge - in field tests, no increase in the impact factor occurred when more than one truck was on the bridge; in analytical studies the impact factor when more than one vehicle was on the bridge was up to four times larger than the impact factor for one vehicle on the bridge; the first conclusion is more realistic since the results were from an actual bridge test and since the second was based on analysis simplifications.

      4) Truck natural frequency - a typical truck has a natural frequency in the range of 2 to 3.5 hertz; this frequency was normally determined by driving a truck over an obstacle and measuring the free vibrations.

      5) Vehicle speed - in the test runs, the vehicle was driven at a constant velocity at speeds ranging from 5 to 50 miles per hour; the

impact factor was proportional to the vehicle speed.

6) Eccentrically driven vehicle - if the vehicle was driven eccentrically across the bridge, the torsional modes of vibration were excited on the bridge.

7) Initial vehicle oscillation - those resulted in large initial dynamic effects.

8) Vehicle weight - as the vehicle weight increased, the amplitude of vibration of the bridge decreased.

9) Vehicle suspension - vehicles with cloverleaf suspensions caused a higher impact factor than vehicles with air suspensions.

10) Parked truck on bridge during testing - had no effect on the dynamic behavior of the bridge.

11) Vehicle driven over an obstacle - caused a 3 to 3.5 times increase in the dynamic response.

12) Vehicles driven side by side - resulted in reduced damping.

b. Bridge Factors:

1) Pavement Roughness - deck roughness causes a slight increase in the impact factor; approach roughness is a major cause of bridge vibrations, with the response for a rough approach as much as three times the response for a smooth approach.

2) Bridge Natural Frequency - if the bridge is in the frequency range of 2 to 5 hertz (that is, approximately equal to the vehicle's natural frequency) the impact factor increases; the first bending mode and the first torsional mode of vibration of the bridge dominate the bridge's dynamic behavior; the loaded bridge frequency is only 6% different from the unloaded bridge frequency.

3) Bridge damping - it is relatively small, typically 0.01 to 0.06; composite bridges have greater damping than noncomposite bridges; simple span bridges have greater damping than multispan bridges.

4) Span length - jerk (derivative of acceleration) values vary inversely with span length.

5) Impact factor - the largest values ranged from 0.30 to 0.85 with most below 0.50.

6) Continuous versus simple span - for bridges with equal span lengths, one with three spans had 1.5 times the acceleration response of one with two spans; however, the highest values of acceleration occurred in simple span bridges; the values of the deflections and the accelerations are less for continuous span bridges than for simple span bridges.

7) Exterior and interior spans - exterior spans have greater vibration amplitudes than interior spans.

8) Span-flexibiliby - increasing bridge flexibility increases the duration of vibrations, i.e. decreases the damping of the bridge and makes the bridge oscillate for longer periods; the amplitude decreased as flexibility decreased.

9) Two-dimensional behavior - mode shapes show that the bridge deck has two-dimensional behavior, with flexural in the longitudinal and torsional in the transverse directions.

10) Higher modes - these had a significant effect on the behavior in analytical studies, but not in field studies; in the analytical studies, at least three modes are needed in the solution for reasonable results; in field studies, it is usually only necessary to study the first mode behavior.

11) Composite behavior - even noncomposite bridges behaved compositely under typical traffic loading, i.e. slip between the steel beam and the concrete slab do not occur until the bridge is loaded near ultimate.

Chapter 3

HUMAN SUSCEPTIBILITY AND DESIGN RECOMMENDATIONS

3.1 INTRODUCTION

Many of the dynamic bridge behavior studies involved reviews of the provisions in design specifications and some resulted in revised recommendations. These are discussed in this chapter. Section 3.2 contains discussions of present code specifications and their adequacy in preventing vibrations. Studies on human susceptibility to vibrations are in Section 3.3. Finally, several studies introduced new limits for use in preventing uncomfortable bridge vibrations and these are summarized in Section 3.4. Susceptibility factors and design limits are summarized in Section 3.5.

3.2 CODE SPECIFICATIONS

A Committee was formed in the middle 1950's to study the deflection limitations of highway bridges (24). They found that the depth-span ratios, deflection limitations, and the impact values were all used in the AASHTO code to limit dynamic behavior. The depth-span ratios originated in the railroad bridge specifications. The deflection limitations were added to the code in the 1930's when it was found that steel highway bridges that had noticeable vibrations typically had static deflections greater than 1/800th of the span. Finally, the impact values came from an envelope of curves for tests done on railroad bridges.

The Committee studied current research on bridges to see if the limits still applied. They found that the typical reason for limits was to avoid structural effects like excessive deflection, fatigue, and undesirable psychological reactions. They listed the factors influencing dynamic behavior. Limiting one particular factor in this list was not sufficient to reduce vibrations. They found that a person's reaction to vibration depended on the amplitude, frequency, acceleration, and jerk, or change in acceleration. It was difficult to set limits since susceptibility to vibrations varied from person to person and from situation to situation. They concluded that more studies were needed and that limits in the code at that time should be left as they were.

Four studies in the 1970's and the 1980's discussed the AASHTO Specifications. The earliest of these studies was by R. N. Wright and W. H. Walker (113). They found that the limits on the depth-span ratio and the deflections led to uneconomical use of high strength steel. This was due to the necessity of extra embankments and tight vertical clearances, needed to get more depth. They conducted an analytical study and found that the limits did not assure human comfort since they did not directly control the dynamic behavior of the bridge. The second study, by M. J. Bartos (8), demonstrated that the AASHTO deflection limits put most medium span steel bridges at a natural frequency of 2.5 hertz, which coincides with the typical truck frequency range of 1.5 to 5 hertz. Since some dynamic bridge studies have found that unacceptable dynamic effects can occur when the natural frequency of the bridge is approximately equal to the natural frequency of a truck, they questioned the current deflection limit. The third study was by J. R. Billing and R. Green (11). They stated that the impact factor in the AASHTO Specification was developed for railroad bridges and locomotives, and that it is incorrect to assume that the factor applies to present day highway bridge structures and vehicles. The final study was by J. T. Gaunt and C. D. Sutton

(37). They found that people were susceptible to the acceleration of displacement rather than the displacement due to vibrations. Therefore, AASHTO specifications may prevent economical use of steel since the code limits for the depth-span ratio and the deflection-span ratio reduce deflections, not accelerations. These code limits have remained unchanged since these papers were written.

Three studies refer to the new Ontario Bridge Code, which was based on fifteen years of bridge testing. In the paper by M. J. Bartos (8), it is noted that the maximum deflection in the Ontario code was reduced to 1/450th of the span to reduce the natural frequency of the medium span bridge to 1.5 hertz which is out of the typical truck frequency range. In addition, in another paper discussing design loads for bridges (75), it was stated that the Ontario code specified raising the impact value if the bridge`s natural frequency was in the 1.0 to 6.0 hertz range. This was to compensate for resonance. The final paper on the new Ontario Bridge Code was by R. A. Dorton and B. Bakht (27). They discuss the dynamic load allowances, which vary for different types of bridges: bridges without sidewalks, bridges with sidewalks and few pedestrians, and bridges with sidewalks and many pedestrians.

Two studies also contain discussions of the British bridge design code. C. W. Brown (14) stated that all bridges, except for very light bridges, were little affected structurally by vibrations, though humans may be bothered by the vibrations. The British code includes impact to cover dynamic affects but does not cover additional surface irregularity induced vibrations. In J. R. Billing and R. Green`s (11) paper written in 1984, the future British bridge design code is discussed. The code will be based on limit state design. Instead of impact values, the code will have dynamic load allowances prescribed as an increase to and a fraction of the prescribed highway live load. As an alternative, dynamic analysis or a field test may be used. Instead of the span-depth and span-deflection limits, the new code will have vibration limits (with three levels depending on the amount of pedestrian usage) based on equivalent deflections at the edge of the structure and average truck weights.

A study by G. P. Tilly, D. W. Cullington, and R. Eyre (90) contains a review of the British specification for footbridges written by the British Standards Institution. The acceleration is limited to one-half the square root of the first bending frequency for frequencies up to 4 hertz. For a bridge with a first bending frequency between 4 and 5 hertz, a reduction factor may be applied to the response of the bridge. Above a frequency of 5 hertz, a bridge is too difficult to excite and vibrations can be ignored. For vandal loading, i.e. a group of people jumping on the bridge, the British code specifies that the bearings should be able to resist upward and lateral movements and that prestressed bridges should be able to carry a 10% reversal of the live load moment. Finally, the British code recommends that in design calculations, a damping value of 0.03 should be used for steel bridges, a value of 0.04 for composite bridges, and a value of 0.05 for concrete bridges.

3.3 HUMAN SUSCEPTIBILITY TO VIBRATIONS

F. Postlethwaite (73) studied human susceptibility to vibration back in the middle 1940`s. He concluded that human beings can stand less vibration than the structure which transmits the vibrations to them. In addition, people who are standing or sitting can tolerate less vibration applied vertically than in any other direction. In his paper, he discussed six previous studies on human perception of vibration including traffic, ship, aircraft, and elevator induced vibrations. Using the data from the studies, he plotted a family of curves of acceleration versus frequency. His family of curves included imperceptible, threshold of perception, uncomfortable and very uncomfortable. In the frequency range of 1 to 6 hertz (typical for highway bridges), the value of acceleration for uncomfortable vibrations was from 5 to 2 feet per second per second, and for strongly noticeable, from 0.5 to 0.6 feet per second per second.

In a more recent study, J. F. Wiss and R. A. Parmelee (112) tested human perception of transient vibrations, i.e. typically induced by people walking in a building. They subjected people, individually, in a standing position to various combinations of frequency (2.5 to 25 hertz), peak amplitude (0.0001 to 0.1 inch), and damping (0.01 to 0.16). Each person rated the vibration on a scale of imperceptible, barely perceptible, distinctly perceptible, strongly perceptible, and severely perceptible. They tested 40 people of a variety of ages, occupations, and of both sexes. They used a vibration generator in a carpeted, sound proofed testing room to test people.

To process their data, Wiss and Parmelee used a statistical analysis with the method of least squares. They found no difference in a person`s perception if they were sitting on a hard chair or if they were standing. They compared their work with one of the studies summarized in Postlethwaite`s paper and found good correlation. In addition, they found that as damping increased, perception decreased. They plotted frequency times displacement versus damping. In the critical damping range of 0.02 to 0.04, typical of highway bridges, the frequency times peak displacement in the distinctly perceptible area ranged from 0.018 to 0.062 cps-inches and in the strongly perceptible area ranged from 0.062 to 0.18. These values were constant for the frequency range they tested.

In the 1970`s, T. M. Murray (70) discussed ways to design floor systems to prevent vibrations. He found that human perception of vibrations depended on the frequency, initial amplitude, and damping of the vibration. He compared two scales that included the effects of damping: Reiher-Miester and Wiss and Parmelee. He used the Reiher and Miester scale since it was based on actual floor system tests, rather than laboratory simulated floor vibrations as in Wiss and Parmelee`s study. He used the strongly perceptible region scale. This limits amplitudes to 0.15 to 0.025 inches of displacement for the frequency range of 1 to 6 hertz. From the scale he derived equations to estimate the frequency and the amplitude of vibrations in T-beam floors with slab deck surfaces.

3.4 SUGGESTED DESIGN LIMITS

In conjunction with a field test in the 1950`s on a three span continuous bridge, J. M. Hayes and J. A. Sbarounis (48) presented several design recommendations. They based their recommendations on a simplified equation for determining the frequency of a bridge. If the bridge`s frequency was approximately equal to the truck`s frequency, they recommended increasing the bridge stiffness or adding shear connectors between the slab and stringers to reduce the bridge`s dynamic behavior.

In their paper on design provisions for dynamic loading, J. R. Billing and R. Green (11) made several recommendations. They suggested that the value of impact should be equal to 0.15 plus the speed parameter, equal to the vehicle velocity divided by the quantity of two times the length of the span times the natural frequency of the bridge. During a field study done in 1980, human response was also measured. The range of accelerations for response was 0.015 to 0.025 g for slightly perceptible, 0.052 g for distinctly perceptible, and 0.076 g for strongly perceptible.

In the early 1970`s, R. N. Wright and W. H. Walker (113) made several recommendations based on an analytical study. For the dynamic response, they recommended that the whole bridge response be modeled as a unit at any cross section and that distribution factors be used to account for the transversely nonuniform moments, shears, and deflections. They gave a new formula for the impact, applicable for both simple and continuous spans: impact is equal to the speed parameter plus 0.15. The speed parameter is equal to the vehicle speed divided by the quantity of 2 times the fundamental frequency of the bridge times the length of the span. In addition, they gave a simplified equation for calculating the bridge frequency.

Wright and Walker also studied human reaction to vibrations due to the vertical motion of the bridge. They found it is only noticed by people when they are walking or standing still. The human response is subjective and varied. For the range of frequency of bridge vibration, the acceleration of the vibration was what people noticed the most. People were less susceptible to vibrations that decayed rapidly, so they recommended neglecting the static component of deflection. From their study, they suggested limiting the amplitude of the dynamic component of acceleration in the fundamental mode to 100 inches/second/second.

Two studies done in the middle of the 1970`s contain design recommendations. J. Blanchard, B. L. Davies, and J. W. Smith (12) studied dynamic behavior in footbridges. They found that footbridges which are structurally sound, often have vibrations which are felt as unacceptable by people. They limited the peak acceleration due to vertical vibrations (in the units of meters/second/second) to one half the square root of the natural frequency of the bridge. If the first mode frequency of the bridge was greater than 5 hertz, they recommended ignoring vibrations. If the first mode frequency of the bridge was in the 4 to 5 hertz range, they recommended using dampers or other means to reduce the response. Finally, they found that abnormal loads from crowds or vandals were too hard to quantify and too rare an occurrence, so they recommend neglecting them. At about the same time, R. Eyre and I. J. Smith (31) recommended limiting the amplitude of vibration for deflection to less then plus or minus 2.5 millimeters for steel box girder bridges.

D. G. Manning (63) in the early 1980`s made several recommendations for bridge design in his paper on bridge deck repairs. He found that the maximum dynamic response occurred when the vehicle speed was such that the time to cross the span was equal to the fundamental period of the bridge. He concluded that this maximum response would never occur in spans greater than 50 feet long, since the vehicle speed would be too high. He stated in his paper that the human body was a dynamic system capable of sensing very small levels of vibrations. He recommended that the vibrational velocity amplitude be no greater than 0.2 inches/second.

In J. T. Gaunt and C. D. Sutton's (37) study of bridge vibrations, they found that for frequencies in the range of a typical bridge's natural frequency, the human body was sensitive to the derivatives of displacement rather than the displacement. For the frequency range of 1 to 6 hertz, people were most susceptible to the jerk value (the first derivative of the acceleration). For the frequency range of 6 to 20 hertz, people were most susceptible to the acceleration, and for the 20 to 60 hertz range, people were most susceptible to the velocity. Bridge damping decreased the sensitivity.

Gaunt and Sutton suggested that the acceleration should be limited to 100 inches/second/second as recommended by Wright and Walker (113). Only five of the bridges tested by Gaunt and Sutton surpassed this limit. Using the simplified equations found in Wright and Walker's work and the acceleration defined as equal to the impact times the quantity squared of 2 times 3.1416 times the fundamental frequency of the bridge, they found that these values generally agreed with their field results.

G. P. Tilly, D. W. Cullington, and R. Eyre (90) presented several design suggestions in their article on the dynamic behavior of footbridges. Typically, they found that humans were disturbed by vibrations long before the bridge was structurally unsound. Therefore, bridge vibration limits were typically based on human comfort. They recommended that the bridge stiffness be greater than 8 kN/mm, that the damping be greater than 0.03 logarithmic decrement, that the bridge's fundamental frequency be outside the range of 1.7 to 2.2 hertz, and that span lengths should be less than 20 meters for concrete bridges and less than 35 meters for steel bridges. They found that if typically two or more of these limits are violated, the bridge will vibrate excessively. To decrease the dynamic behavior of a bridge already built, they recommended increasing the stiffness or adding dampers or dynamic absorbers.

3.5 SUMMARY

1. Human susceptibility is influenced by:

   a. Derivatives of Deflection - typically humans perceive the derivations of deflection, i.e. the velocity and acceleration, rather than deflection:

      1) Jerk (derivative of acceleration) - most perceivable of the deflection derivatives in the 1 to 6 hertz frequency range.

      2) Acceleration - most perceivable of the deflection derivatives in the 6 to 20 hertz frequency range.

      3) Velocity - most perceivable of the deflection derivatives in the 20 to 60 hertz frequency range.

   b. Damping - reduces human perception of vibrations.

   c. Overall Response -

      1) Is subjective and varied.

      2) Vibrations are only noticeable to people when they are walking or standing still.

   d. People are most affected by vibrations if the static deflections of the bridge are large and if the frequency of the bridge is high.

2. The ranges of perception are:

   a. Damping of 0.02 to 0.04:

      1) Distinctly Perceptible: 0.018 to 0.062 cps-inches.

      2) Strongly Perceptible: 0.062 to 0.18 cps-inches.

   b. Frequency of 1 to 6 hertz:

      1) Uncomfortable Vibrations: 5 to 2 feet/sec./sec.

2) Strongly Noticeable: 0.5 to 0.6 feet/sec./sec.

3) Strongly Perceptible: 0.15 to 0.025 inches amplitude.

4) Distinctly Perceptible: 0.052 to 0.076 g.

5) Strongly Perceptible: 0.076 g and over.

3. Present Limits for Vibration -

a. In the AASHTO Specification design for vibrations is treated by:

1) Use of impact factors.

2) Limiting the deflection-span ratio.

3) Limiting the depth-span ratio.

4. Proposed Limits for Vibration include:

a. Acceleration:

1) Limit the dynamic component of acceleration in the first mode to 100 inches/sec./sec.

2) Limit the peak acceleration to one-half the square root of the fundamental frequency of the bridge.

b. Amplitude of Vibration: keep less than 2.5 mm.

c. Velocity: limit the vibration velocity amplitude to less than 0.2 inches/sec.

CHAPTER 4

DYNAMIC STUDIES OF THE FOUNDERS BRIDGE

4.1 INTRODUCTION

This chapter contains the results of an initial study based on the dynamic behavior. The STRUDL program was used to determine the free vibrational behavior, i.e. the natural frequencies and mode shapes associated with free vibrations. No attempt was made to incorporate a moving load effect since this involves a highly sophisticated solution scheme, well beyond conventional programs like STRUDL (this work is planned for the second part of this investigation).

4.2 ONE DIMENSION MODEL

The equivalence beam method was used to model the Founders Bridge, which simplifies the plate girder bridge into a one dimensional continuous beam model. The girder haunches were modeled by equivalent homogeneous sections. Figure 1 shows this model, based on two different cross sections. Segments 3, 4, 7, 8, 11, and 12 have the cross sectional areas equal to 1482.4 in-2 and moments of inertia equal to 1082620.1 in-4 and the other elements have an area of 1467.4 in-2 and a moment of inertia of 785500.0 in.4.

The STRUDL program was used to determine the first five natural frequencies and mode shapes of the girders. Both a lumped mass model and a consistent mass model were used. The material used in the analysis was as follows. For the steel girders, the modulus of elasticity was 29,000 ksi, and the density of the material was 500 lb per cubic feet. For the reinforced

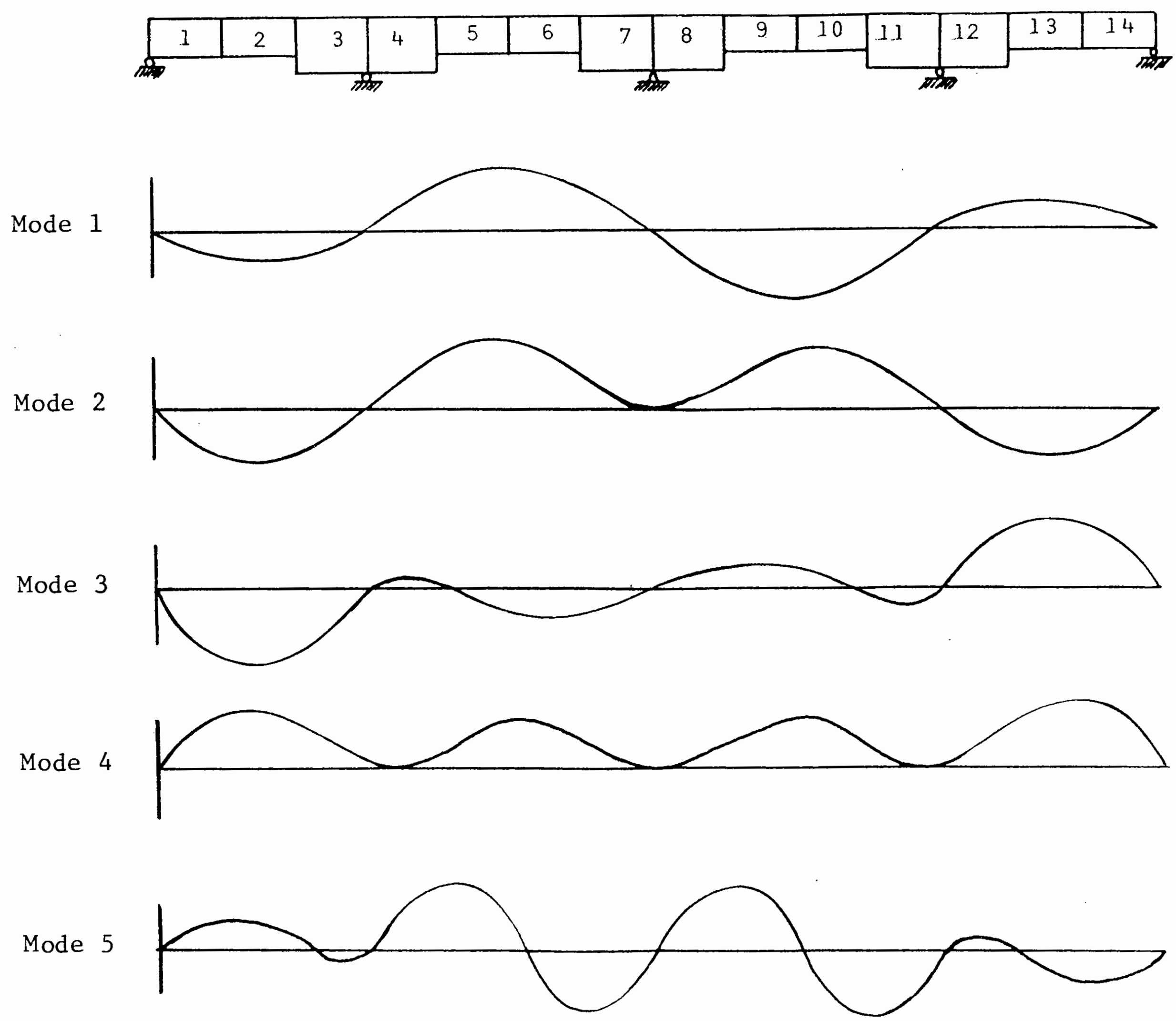

Figure 1 Model and Mode Shapes for One-Dimensional Analysis

concrete slab, the modulus of elasticity was 3000 ksi. The weight of the floor beam, the slab and the girder were included in calculations. The results for the girders are listed in Table 1 using a lumped mass model and in Table 2 using a consistent mass model. The natural frequencies of the floor beams (i.e. the beams perpendicular to the main longitudinal girders) are listed in Table 3. The first five mode shapes are shown in the Figure 1.

4.3 TWO DIMENSIONAL MODEL

A dynamic study using STRUDL was attempted for a two-dimensional grid model of the bridge. The first five natural frequencies and mode shapes were to be found. It was found that the computer did not converge to a solution during the specified time limit. Thus, it was too costly to run this two-dimensional analysis, and this part of the study was abandoned.

A full two-dimensional analysis has been proposed for the subsequent study. The limited dynamic study has shown that the bridge falls in the typical range of natural frequencies for steel girder bridges.

TABLE 1 NATURAL FREQUENCIES OF THE LUMPED MASS MODEL

Mode	Frequency (cps)	Period (seconds)
1	2.14094	0.467085
2	3.06404	0.326367
3	4.41642	0.226428
4	4.84424	0.206431
5	7.82111	0.127859

TABLE 2 NATURAL FREQUENCIES OF THE CONSISTENT MASS MODEL

Mode	Frequency (cps)	Period (seconds)
1	2.19626	0.455320
2	3.16756	0.315701
3	4.65532	0.214808
4	5.08326	0.196724
5	8.67384	0.115289

TABLE 3 NATURAL FREQUENCIES OF THE FLOOR BEAMS

Mode	Frequency (cps)	Period (seconds)
1	13.6719	0.0731428
2	20.5040	0.0487709
3	54.6169	0.0183094
4	60.1389	0.0166282
5	127.6860	0.0078317

Chapter 5

CONCLUSIONS

Over 100 papers on bridge vibration were reviewed in the literature search. Included were field studies, model studies, analytical studies, and studies of human susceptibility limits.

A thorough review of the literature has shown that the following are causes of vertical vibrations in bridges:

1. Vehicle speed - increased speed increased the dynamic increment.
2. Pavement roughness - this can be a major factor.
3. Vehicle suspension system - this involves both the springs and tires.
4. Approach condition - this includes the surface roughness on the approach and the joint between the approach and bridge.
5. Eccentricity of the load - torsional vibration modes result if this is large.
6. Natural Frequency - the relation between the natural frequency of the bridge and that of the vehicle is related to the magnitude of the dynamic increment in the deflections.
7. Bridge structure - the type of structural members and the layout are directly related to the overall behavior.

It has been found that large amplitudes occur when the natural frequencies of the bridge and vehicle are close and when the bridge has large static deflections. People most notice vibrations when the amplitudes are large and the bridge natural frequencies are high.

Studies have shown that the fundamental flexural and torsional vibration modes control the behavior and that field studies usually need only deal with these. The vibrational behavior is best measured with the attachment of accelerometers to the bridge at or near the midspans. These should be put on both sides so that both the torsional and flexural modes can be measured. In addition to the direct measurement of the accelerations, the velocity and deflections can be obtained through integration. A spectral analysis is used to determine the character of the frequency content.

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