Final Report

STATISTICAL ANALYSIS APPLIED TO MANAGEMENT DECISIONS

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TABLE OF CONTENTS

Introduction 1

Procedure 2

References 12

Appendix 13

INTRODUCTION

The purpose of quality control is to assure the purchaser that all material incorporated into construction meets or exceeds a pre-determined measure of quality. The engineer desires such assurance to assist in establishing confidence that the completed construction has the strength and durability expected by the designer. The administrator uses the same assurance to establish that the purchaser received what was requested and paid for.

The materials specialist develops the testing methods, the designers and engineers establish the predetermined level of performance specified, and the administrator determines the level of resources devoted to enforcement of specifications. The policy decision concerning resources should be directly related to a balance between cost of specification enforcement and potential damages brought about by incorporation of sub-specification material. This trade off can only be established through extensive compilation of experience. Day by day decisions must then be made by management to utilize the resources available to enforce specifications.

Considering bituminous concrete as a typical material, it is obvious that the Materials Testing Section of the Connecticut Department of Transportation can not have an unlimited number of inspectors on hand. There are some 40 bituminous concrete plants in the state of which during a typical summer day 15 to 20 may produce material for the state. However many days would see only 10 or 13 plants producing. Costwise, it is realistic to have a sufficient number of inspectors to cover a number of plants less than the maximum number. It is thus necessary to develop a procedure for assignment of inspectors on peak days.

The manager of the inspectors can be expected to take into account such factors as tonnage, rejection rate, experience with each plant, etc., when assigning inspectors. As the number of operating plants increases, these assignment decisions become more difficult. To assist the manager with this problem, a scheme was devised, based on the impartiality of statistics, to analyze plant-test reports of material produced at various mix plants. The result of the analysis is the development of one constant per mix-plant, representing the quality of the material produced at that plant, on which the management decision can be based.

Statistical theories are instrumental in transforming the data from plant-test reports into a usable form. Selected values such as averages and standard deviations are representative quantities of each material. Decisions concerning the assignment of state inspectors can be based on these statistical indicators.

It is important to recognize the limitations of such an approach. Should the amount of data included in the analysis fall below that anticipated by the statistical procedure, decisions based on the analysis will be questionable. It is also assumed that the data collected from the plant test reports is representative of the material produced. Reasonably current data should be used in the analysis so that the results of the analysis have a pertinent bearing on the decisions at hand. The results of the analysis are only an aid for the decision-making process. The engineer in charge must evaluate the situation at hand and make the ultimate management decision.

PROCEDURE

The statistical procedure itself involves a small number of basic concepts. Each class of pavement mix has specifications pertaining to gradation and bitumen content. Producers who supply these mixes to the state must meet these specifications within certain tolerance limits. The range of these tolerance limits is the subject of the statistical analysis.

All plants have a job mix formula for each mix that is produced for the state. Each job mix formula specifies the gradation and bitumen content which the plant must meet. The job mix formula varies slightly from plant to plant but all plant job mix formulas are within the state's master range. How often a plant meets its own job mix formula and the degree by which it may vary from its job mix formula are measures of the plant's production performance.

The test reports list the sieve analysis and the percent bitumen for each sample tested. To offset the problem of some plants having acquired more data than other plants, only data from the last test day should be used. The required calculations for ranking should be carried out at time intervals not exceeding two weeks. Plant-mixes not produced within that period would not be ranked.

For a given plant and mix, the standard deviations are found for the bitumen content and each sieve size reported. These standard deviations are then adjusted according to the number of tests used to compute the deviations. As the job mix tolerances specified by the state are assumed to represent ± 3 sigma (range of deviation for essentially 100% confidence), the adjusted standard deviations are multiplied by 3 and then divided by the respective job mix tolerances. Division by the job mix tolerances normalizes, or brings to a common base, the variation of bitumen content and various sieve sizes. The normalized deviations are then multiplied by weighting factors. The bitumen content and/or sieve sizes which are more crucial in the production of the mix can be given greater importance in the ultimate ranking by increasing their weighting factors.

The immediate goal is to rank the various mix plants according to how consistently they produce a given mix. To accomplish this, all the weighted deviations for the parts of a mix are summed and this total is divided by the sum of the weighting factors used for that mix. This last division allows for a comparison to be made among all the different mixes produced. The quotient obtained is the plant's rank for the mix. A large rank number is an indication of poor quality control for the particular mix.

It should be noted that the standard deviations for each sieve and bitumen content for a mix are centered about the production average rather than the plant's job mix formula. The possibility exists that a plant can have a small deviation about an average which is out of the master range limits. This plant, according to the suggested ranking scheme, would still be ranked favorably when compared to the other plants. However if a plant has good control over its production process this situation is not likely to occur. A plant which would allow its production averages on any sieve or bitumen content to fall outside the master range is also likely to have large deviations. Consequently, this plant would be ranked poorly.

A statewide sequential listing of the plant-mixes can be made based on the ranking number each plant has for each of the mixes it produces. This listing should be revised every two weeks and used when deciding where to send state inspectors. Plants which are placed high on the list for a particular mix should be inspected more frequently than plants which appear lower on the list. Regardless of how a plant is ranked, each should be inspected periodically. This is to insure that data necessary for the next revision of the ranking is collected from all the plants. A plant which has no data collected for a mix it produces cannot be incorporated into the ranking process.

An example demonstrates the statistical analysis and ranking procedure discussed above. Two plants, A and B, will be used for the example with both

Then the standard deviation

	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
4 - 3.6	.4	.16
3 - 3.6	-.6	.36
3 - 3.6	-.6	.36
4 - 3.6	.4	.16
4 - 3.6	.4	.16

$$1.2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\sigma_A \sqrt{\frac{1.2}{5}} = 0.49$$

The procedure is repeated for Plant B, Tests 6 → 8

$5 + 6 + 6 = 17$

AVG = $17/3 = 5.67 = \bar{x}$

		$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
5 - 5.67	=	-.67	.449
6 - 5.67	=	.33	.109
6 - 5.67	=	.33	.109

$$.667 = \sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\sigma_B = \sqrt{\frac{.667}{3}} = 0.47$$

Since the number of tests are small, a correction factor is applied to the deviations.

$$\sigma' = \frac{\sigma}{c} \qquad c = \text{correction factor}$$

For n = 5 → c = .8407 (see Table 1)

For n = 3 → c = .7236

Therefore $\sigma_A' = \frac{.49}{.8407} = .583$

$$\sigma_B' = \frac{.47}{.7236} = .650$$

plants producing the same mix. Plant A had 5 test reports on the last day tested and Plant B had 3. All the data for the #200 sieve for a two week period is listed below. Detailed computations for the #200 sieve are shown below and are typical for all the sieve sizes and bitumen content.

TEST	PLANT A % Passing #200 Sieve		PLANT B % Passing #200 Sieve	
1	4		7	
2	3		5	
3	4		6	
4	4		6	
5	4	↑	6	
6	4	Last test day	6	↑ Last test day
7	3		6	
8	3		5	↓
9	4	↓		

1) Compute the standard deviation (σ) for the tests done on the last test day in the test period.

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n}}$$

where n = # of tests

x_i = individual test numbers

$\overline{x}$ = average of test numbers

PLANT A Tests 5 → 9

First find the average $(\Sigma x_i)/n$

$$4 + 3 + 3 + 4 + 4 = 18$$

$$\text{AVG.} = \frac{18}{5} = 3.6 = \overline{x}$$

2) The corrected deviations are now normalized using the Job Mix Tolerances. For Class 2 on the #200 sieve, the tolerance is 2% ± (see Connecticut Standard Specifications, Form 811). $3\sigma_u$ then is assumed equal to 2% so that $\sigma_u = 2/3 = .66$. σ_u is a unit deviation and σ^n is the normalized deviation.

$$\sigma^n = \frac{\sigma'}{\sigma_u}$$

$$\sigma^n_A = \frac{.583}{.66} = .8745 \qquad \sigma^n_B = \frac{.65}{.66} = .975$$

The preceding computations are repeated for the bitumen content and other sieve sizes.

3) Weighting factors are applied to express the importance of a particular sieve size of bitumen content in the production of the mix.

SIEVE SIZE	WEIGHTING FACTOR (ASSUMED)
200	1.5
50	1
8	1
4	1
3/8	1
1/2	1
3/4	1
BIT.	2.0

$$r = w \times \sigma^n$$

An r value is computed for the bitumen content and each sieve size used in producing the mix. The rank of a plant for the mix, R, is the sum of all the r values for the mix divided by the sum of the weighting factors. For the #200 sieve:

PLANT A	PLANT B
$r = 1.5\ (.8745) = 1.31$	$r = 1.5\ (.975) = 1.46$

All the above computations are listed for the remaining sieve sizes and bitumen content for a class 2 mix, for Plants A & B.

PLANT A

Sieve Size	$\bar{x}$	σ	σ'	Sieve Tolerance	σ^n	w	r
200	3.6	0.49	0.583	± 2%	0.8745	1.5	1.31
50	21.4	1.02	1.21	± 4%	0.91	1.0	0.91
8	55.2	1.33	1.58	± 4%	1.18	1.0	1.18
4	64.8	1.94	2.31	± 5%	1.38	1.0	1.38
3/8	95.8	2.31	2.75	± 5%	1.65	1.0	1.65
BIT.	6.34	0.102	0.121	±.5%	0.726	2.0	1.45
						7.5	7.88

$$R_A = \frac{7.88}{7.5} = 1.05$$

PLANT B

Sieve Size	$\bar{x}$	σ	σ'	Sieve Tolerance	σ^n	w	r
200	5.67	0.47	0.65	± 2%	0.975	1.5	1.46
50	17.67	0.47	0.65	± 4%	0.489	1.0	0.489
8	44.0	0.82	1.13	± 4%	0.848	1.0	0.848
4	64.3	1.25	1.72	± 5%	1.03	1.0	1.03
3/8	98.67	0.47	0.65	± 5%	0.39	1.0	0.39
BIT.	6.33	0.047	0.065	±.5%	0.39	2.0	0.782
						7.5	5.00

$$R_B = \frac{5.0}{7.5} = 0.67$$

Since $R_A > R_B$, Plant A will be above Plant B on the list of plants to be inspected.

Although job mix tolerances are specified for each class mix used by the state, in the past these mix tolerances have been arbitrarily chosen. Having made the effort to compile data for the plant ranking procedure it would seem worthwhile to make further use of the information and check how reasonable the specified job mix tolerances are. Again, a statistical analysis is used to accomplish this task. Mix deviations are computed based solely upon the data collected from the plant test reports. It is assumed that these mix deviations will be computed at the end of two or three construction seasons. A comparison between these computed mix deviations and the state specified mix tolerances can then be made.

The procedure used to calculate the job mix to tolerances again only involves basic statistics. For a given class mix, the test reports from all the plants producing that mix are grouped together. Adjusted standard deviations for each of the sieve sizes and bitumen content used in that mix are computed. These deviations determine the limits within which all the plants making that mix were able to produce. Present job mix tolerances should be comparable with these adjusted deviations. An example of computations for the #200 sieve follows.

	PLANT A	PLANT B
	#200 Sieve	#200 Sieve
Test	% Passing	% Passing
1	4	7
2	3	5
3	4	6
4	4	6
5	4	6
6	4	6
7	3	6
8	3	5
9	4	

1) The deviations for the #200 sieve for plants A & B are found and the correction factors applied.

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$$

$\sigma_A = .47$ $\sigma_B = .60$

For n = 9, C = .9137 (See Table 1)

For n = 8, C = .9027

$$\sigma_A' = \frac{.47}{.9139} = .516 \qquad \sigma_B' = \frac{.60}{.7027} = .664$$

2) The deviations are normalized by dividing each deviation by its respective average.

$$\sigma_A^n = \frac{\sigma_A'}{\bar{x}_A} = \frac{.516}{3.67} = .143 \qquad \sigma_B^n = \frac{\sigma_B'}{\bar{x}_B} = \frac{.664}{5.875} = .114$$

3) The normalized deviations are squared and then multiplied by the respective number of test values used in the computation.

$$D_A = (\sigma_A^n)^2(9) \qquad D_B = (\sigma_B^n)^2(8)$$

$$D_A = (.143)^2(9) = .184 \qquad D_B = (.114)^2(8) = .104$$

4) Sum the D value for each plant and then divide that sum by the total number of tests.

$$D_P = \frac{.184 + .104}{9 + 8} = .0169$$

5) The average deviation for a mix is the square root of the D_P value multiplied by the average % passing for the #200 sieve from the state master range chart for the particular mix.

$$\overline{D} = 5 \sqrt{D_P} \qquad \text{(For class 2 mix)}$$

$$\overline{D} = 5 \sqrt{.0169} = .65$$

.65 represents the standard deviation of the population. Therefore, for the data used, the tolerance which the plants produced within was

$$3\sigma = 3(.65) = \pm 1.95$$

meaning that 99% of the time the plants produced within a range of ± 1.95% passing #200 sieve about their respective averages. This compares well with the state job mix tolerance of ± 2%.

REFERENCES

1. Grant, Eugene L., Statistical Quality Control, McGraw-Hill, New York, N.Y., p. 533, 1946.

2. Willenbrock, Jack H., Statistical Quality Control of Highway Construction Material, Vols. I & II, U.S. Department of Transportation, Federal Highway Administration, January 1976.

APPENDIX

TABLE I

FACTORS FOR ESTIMATING σ' FROM $\overline{R}$ OR $\overline{\sigma}$ [1]

Number of observations in subgroup n	Factor for estimate from $\overline{R}$ $d_2 = \overline{R}/\sigma'$	Factor for estimate from $\overline{\sigma}$ $c_2 = \overline{\sigma}/\sigma'$
2	1.128	0.5642
3	1.693	0.7236
4	2.059	0.7979
5	2.326	0.8407
6	2.534	0.8686
7	2.704	0.8882
8	2.847	0.9027
9	2.970	0.9139
10	3.078	0.9227
11	3.173	0.9300
12	3.258	0.9359
13	3.336	0.9410
14	3.407	0.9453
15	3.472	0.9490
16	3.532	0.9523
17	3.588	0.9551
18	3.640	0.9577
19	3.689	0.9599
20	3.735	0.9619
21	3.778	0.9638
22	3.819	0.9655
23	3.858	0.9670
24	3.895	0.9684
25	3.931	0.9697
30	4.086	0.9748
35	4.213	0.9784
40	4.322	0.9811
45	4.415	0.9832
50	4.498	0.9849
55	4.572	0.9863
60	4.639	0.9874
65	4.699	0.9884
70	4.755	0.9892
75	4.806	0.9900
80	4.854	0.9906
85	4.898	0.9912
90	4.939	0.9916
95	4.978	0.9921
100	5.015	0.9925

Estimate of $\sigma' = \overline{R}/d_2$ or $\overline{\sigma}/c_2$.

These factors assume sampling from a normal universe.

1) See Reference 1.