Analysis of Thermally Loaded Laminated Circular Plates

Report No. 3

John F. Carney III, Associate Professor

JHRAC Project 70-2 JHR 72-50

March 1972

ANALYSIS OF THERMALLY LOADED LAMINATED CIRCULAR PLATES

Abstract

This report summarizes the work done on the three-layered phase of the study on thermally loaded laminated circular plates. In this investigation, the system is assumed to be composed of three layers each with its corresponding modulus of elasticity, Poisson's ratio, and coefficient of thermal expansion. The variation of modulus of elasticity with depth of pavement is taken as a parabolic function of temperature. As in the two layered system investigation two extremes of temperature distribution are considered. In the first case, the temperature distribution decreases with depth of pavement, as would be the case at midday. Then, the temperature field is increased with depth to simulate the condition in the early morning hours. The critical stress in the pavement is again taken to be the maximum tensile stress. In addition, this report compares the "three layer" results with those of the one and two layer systems.

A study of the results shows the effect of the different layer properties on the stress levels. The following trends for the three-layer system are apparent:

1. If the Young's moduli of the respective layers are essentially equal, critical stresses are lower when the coefficients of thermal expansion do not differ significantly from layer to layer.
2. When $E_1 = E_3 \neq E_2$ $(\sigma_{max})_{tensile}$ for a given (t_1, t_2, t_3) is in general insensitive to the value of Young's Modulus in the second layer.

3, When $E_1 = E_3 \neq E_2$ $(\sigma_{max})_{tensile}$ is very sensitive to the makeup of (t_1, t_2, t_3).

4. When $E_1 = E_2 = E_3$ and $\alpha_3 = R\alpha_2 = R^2\alpha_1$, the critical stress is very dependent on the makeup of (t_1, t_2, t_3). For this case, the maximum tensile stress always occurs at the bottom of the bottom layer and is larger in those cases when t_3 is smallest.

5. When $\alpha_1 = \alpha_2 = \alpha_3$, $(\sigma_{max})_{tensile}$ is insensitive to the makeup of (t_1, t_2, t_3) for a wide range of Young's Moduli.

6. Unless all respective layer parameters (α_1, E_2, ν_i) are within 5% of one another, the single layer approximation of a two or three layer system is not recommended as results differ considerably from the multi-layer results.

This work is continuing in the Civil Engineering Department of the University of Connecticut under the sponsorship of the Joint Highway Research Advisory Council of the Connecticut Department of Transportation and the University of Connecticut.

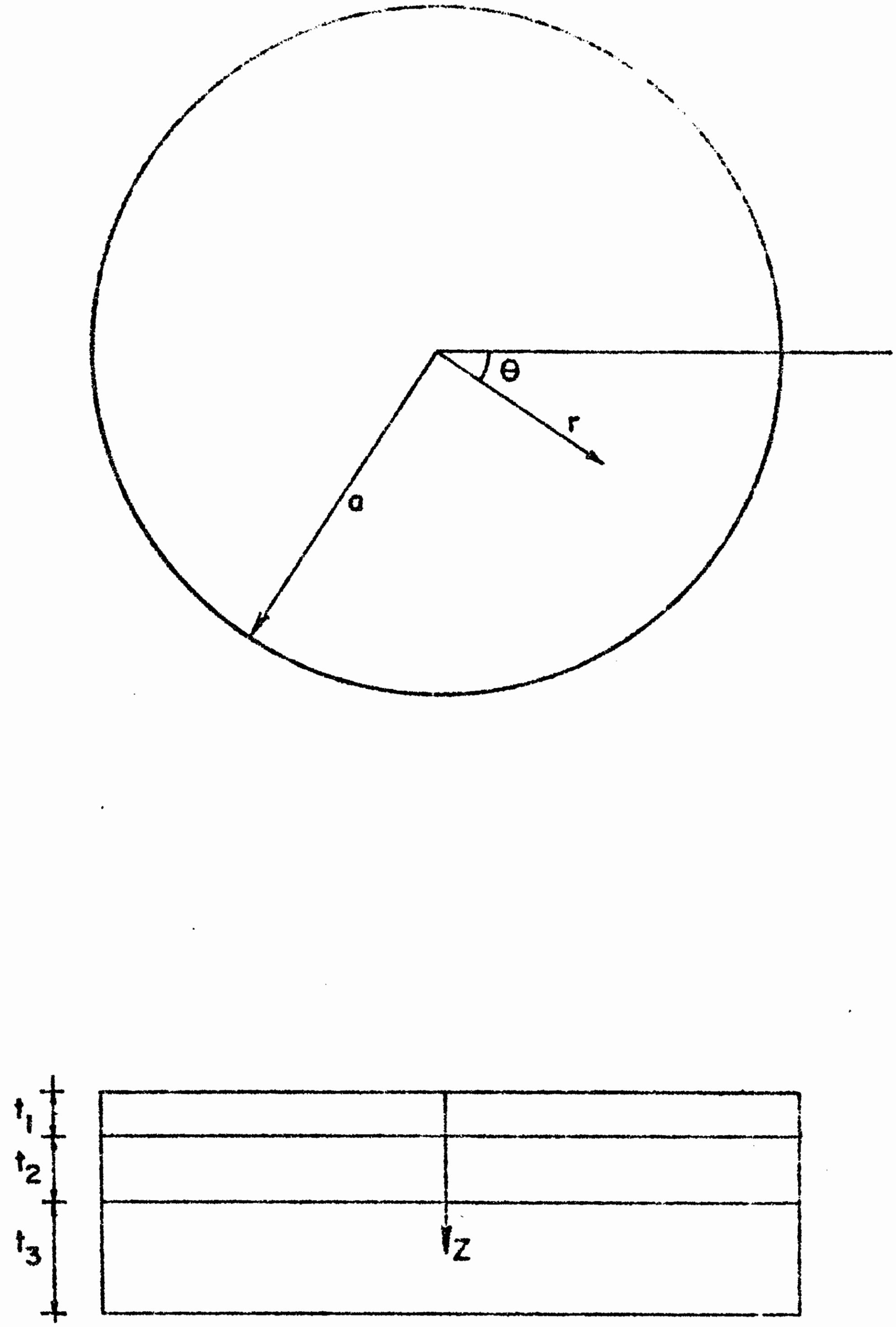

Figure 1 Three Layered Plate System

The remaining development and solution of the problem follows exactly that given in Report No. 1.

Presentation of Results

The addition of the third layer to the pavement system greatly increases the possible parameter combinations which may be studied. These independent parameters include:

a) The three values of Young's modulus, E_1, E_2, E_3.

b) The three coefficients of thermal expansion, α_1, α_2, α_3.

c) The thickness of the individual layers, t_1, t_2, t_3.

d) The temperature distribution, T.

e) Poisson's ratio, ν.

For reasons explained in the first two reports, two different temperature distributions will be considered. In the first case,

$$T = 2.5z + 10 \tag{73a}$$

simulating an early morning in winter condition. In the second case,

$$T = -3.5z + 53.3 \tag{74a}$$

which models the temperature field on a winter afternoon. In addition, for the three-layered investigation, Poisson's ratio is taken as 0.35.

Now that the temperature distributions and Poisson's ratio have been chosen, attention may be directed to the study of how the variation of E_i, α_i and t_i affect the stress levels in the plate system.

In Figs. (2a) and (2b), the coefficients of thermal expansion in layers one and three are set equal ($\alpha_1 = \alpha_3$) and the ratio of α_1/α_2 is varied

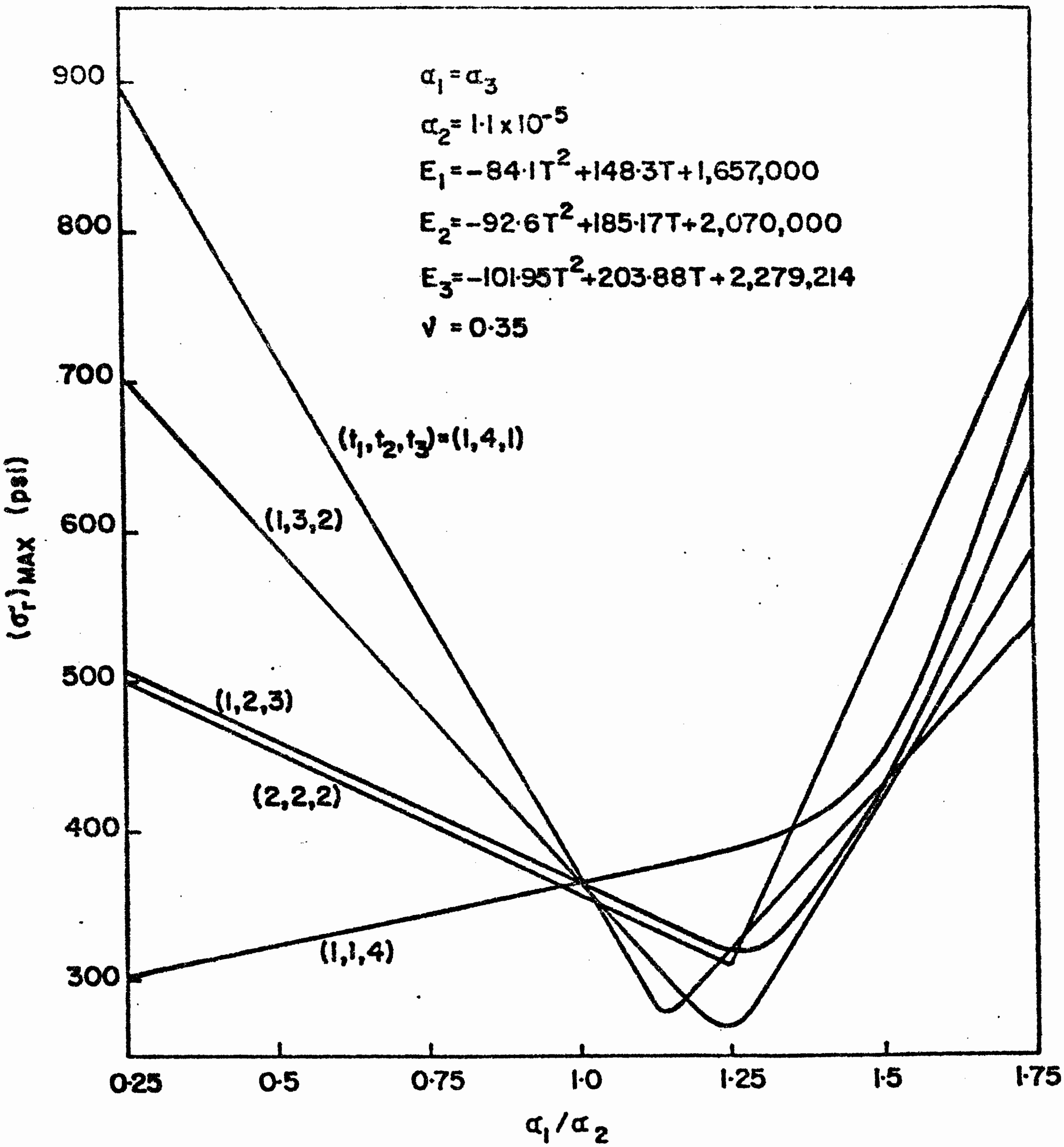

Figure 2a Effect of α_1/α_2 on $(\sigma_r)_{max}$

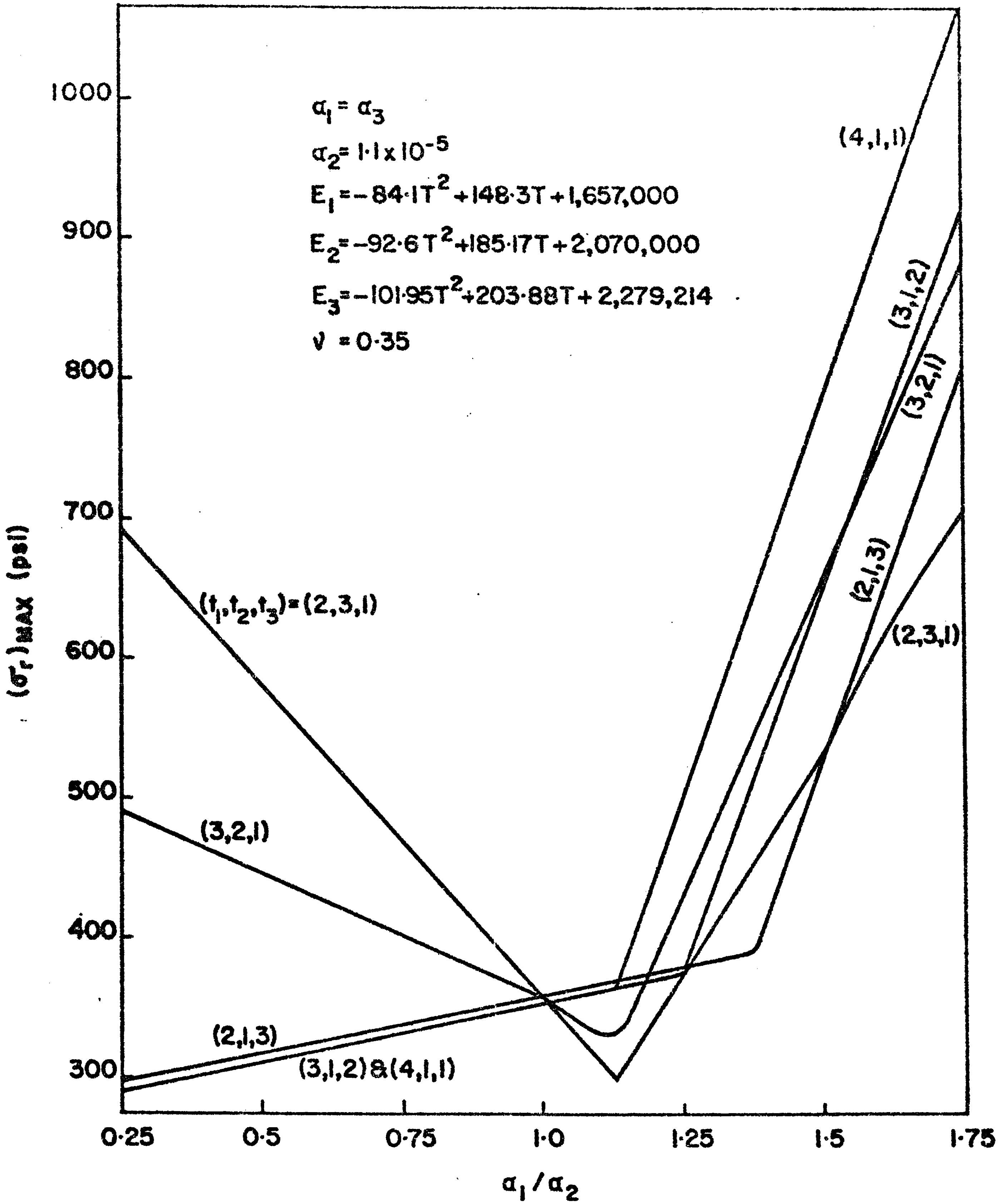

Figure 2b Effect of α_1/α_q on $(\sigma_r)_{max}$

from 0.25 to 1.75 for various values of individual layer thickness ($t_1 + t_2 + t_3 = 6$ in.). Ten different thickness combinations are presented. Note that $(\sigma_{max})_{tensile}$ does not change much as (t_1, t_2, t_3) is varied when $\alpha_1/\alpha_2 = 1$. As one would expect, the maximum values of $(\sigma_{max})_{tensile}$ invariably occur when the coefficients of thermal expansion (α_1 and α_2) diverge from one another. The worst situations occur under the following conditions:

a) $(t_1, t_2, t_3) = (1, 4, 1)$ and $\alpha_1/\alpha_2 = 0.25$

b) $(t_1, t_2, t_3) = (4, 1, 1)$ and $\alpha_1/\alpha_2 = 1.75$

The important fact made clear from a study of Figs. (2a) and (2b) is that, if the values of Young's moduli of the layers are not drastically different from one another, critical stresses are lower when the coefficients of thermal expansion do not vary significantly from layer to layer.

In Figs. (3a) and (3b), $(\sigma_{max})_{tensile}$ is plotted against k_2/k_3, in which k_i is the constant term in the parabolic expression for Young's modulus, which is taken in the form

$$E_i = g_i T^2 + h_i T + k_i \tag{78a}$$

In Fig. (3), the Young's modulus of the top layer is set equal to that of the bottom layer, and the constant component of E_2, (k_2), varied such that $0.25 \leq k_2/k_3 \leq 1.75$. In Fig. (3),

$$\alpha_1 = 1.1 \times 10^{-5} \quad \text{or} \quad 1.4 \times 10^{-5}$$

$$\alpha_2 = 1.3 \times 10^{-5}$$

$$\alpha_3 = 1.4 \times 10^{-5} \quad \text{or} \quad 1.1 \times 10^{-5}$$

depending on whether the temperature distribution increases or decreases with depth. By adopting this approach, the dependence of α_2 on temperature

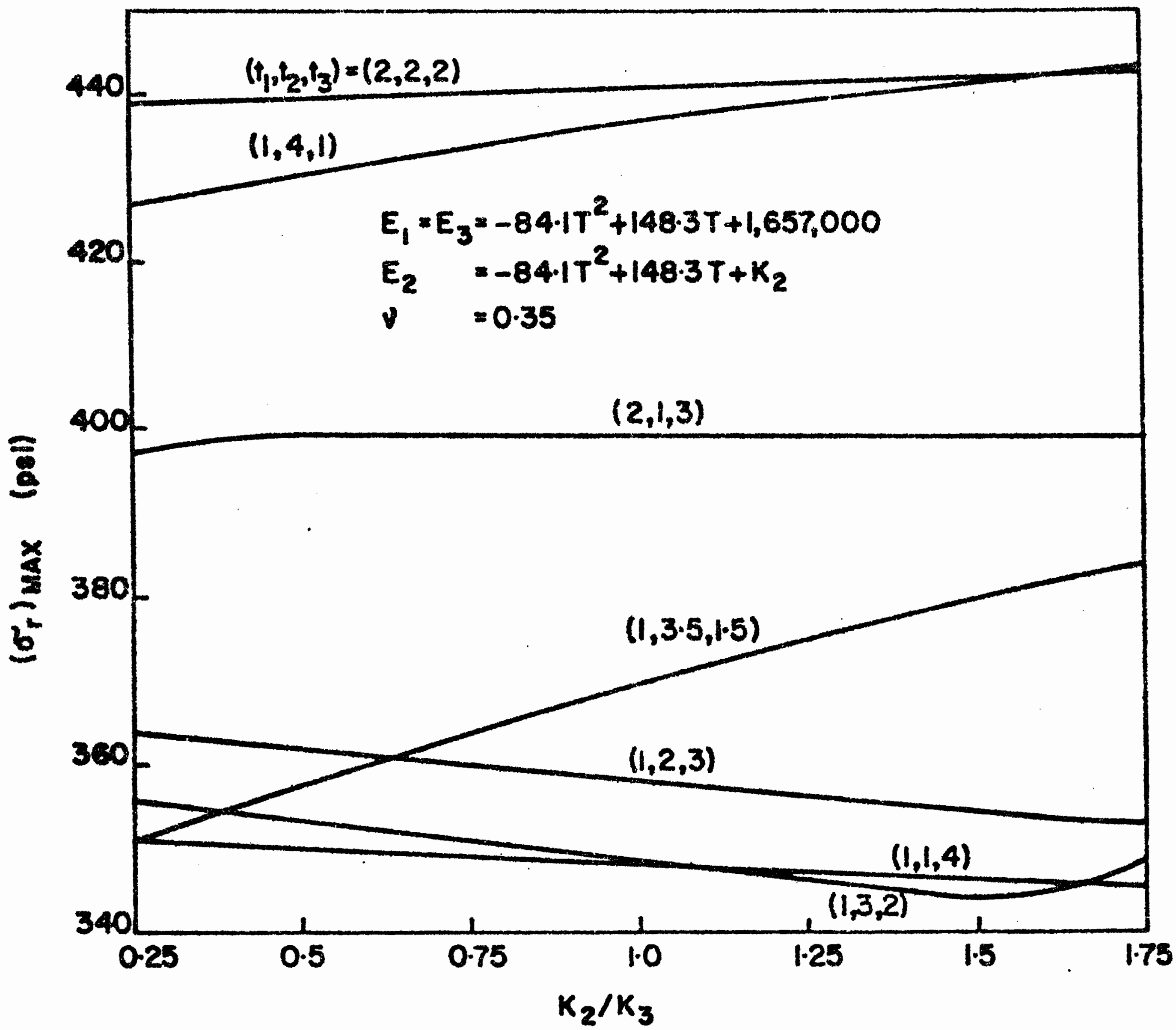

Figure 3a Effect of Young's Modulus on $(\sigma_r)_{max}$

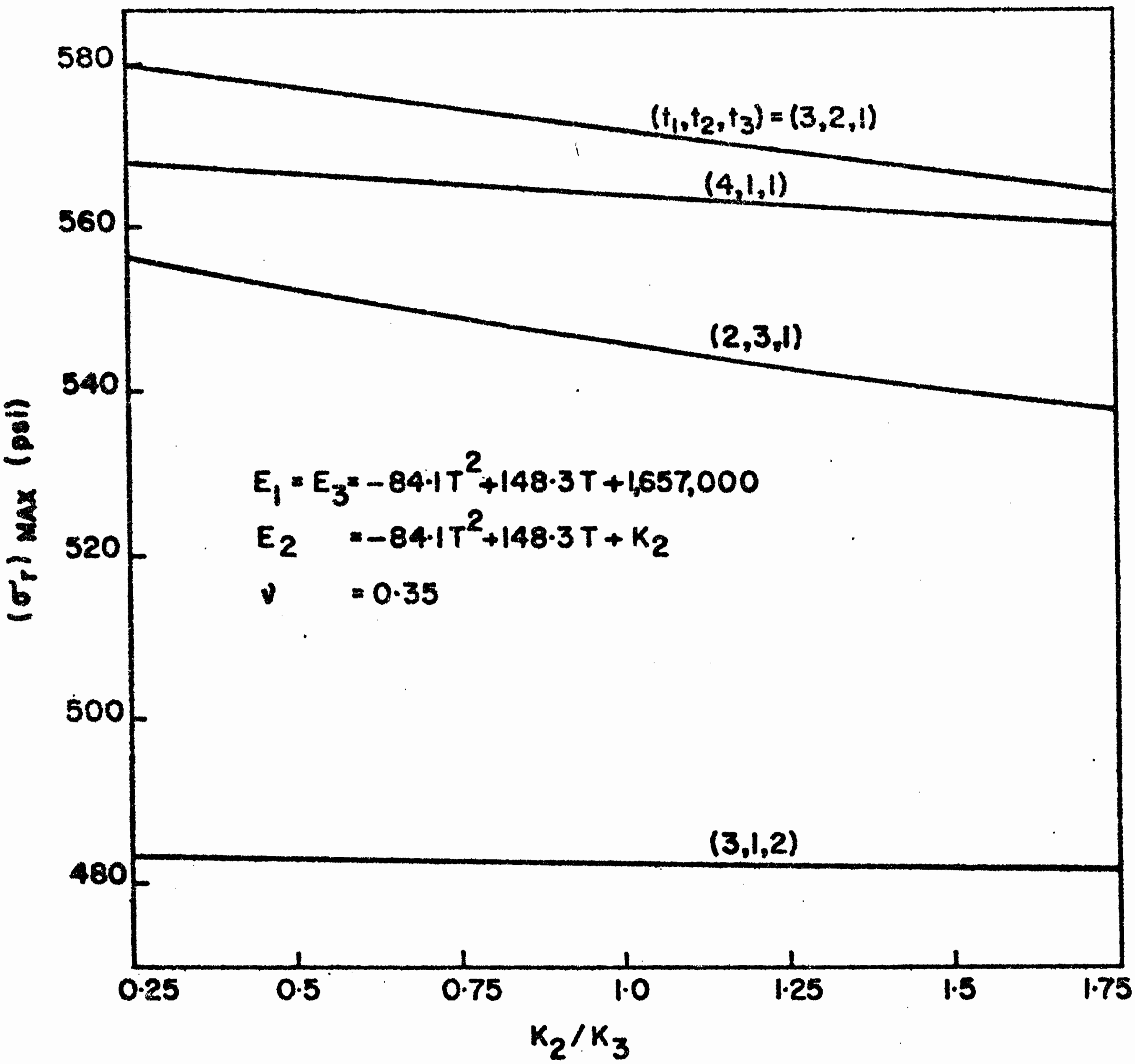

Figure 3b Effect of Young's Modulus on $(\sigma_r)_{max}$

is taken into account. It is interesting to note that $(\sigma_{max})_{tensile}$ for a given (t_1, t_2, t_3) is in general insensitive to the value of Young's modulus in the second layer. This result is quite different in character from that of Fig. 4 (Report No. 2) and is due to the fact that $E_1 = E_3$. On the other hand $(\sigma_{max})_{tensile}$ is quite sensitive to the makeup of the individual layer thicknesses. For example, when $(t_1, t_2, t_3) = (1, 1, 4)$, $(\sigma_{max})_{tensile} =$ 350.83 psi while for $(t_1, t_2, t_3) = (3, 2, 1)$, $(\sigma_{max})_{tensile} = 580.14$ psi.

Figure (4) shows the variation of $(\sigma_{max})_{tensile}$ with the change in coefficients of thermal expansion of the layers. In Fig. (4), the Young's moduli of all three layers are set equal, thereby permitting a study of the effect of α_i (coefficient of thermal expansion) on $(\sigma_{max})_{tensile}$. The following values of α_i were chosen:

$$\alpha_1 = 0.000013 \ /^\circ F$$

$$\alpha_2 = R\alpha_1 \ /^\circ F$$

$$\alpha_3 = R\alpha_2 = R^2\alpha_1 \ /^\circ F$$

When R = 1, the plate degenerates to a single layered one and $(\sigma_{max})_{tensile}$ is independent of the makeup of (t_1, t_2, t_3). For $R < 1$, the critical stress is quite dependent on the relative layer thicknesses. As in Fig. (2), it is demonstrated in Fig. (4) that the stress levels in general rise when the coefficients of thermal expansion differ significantly from layer to layer. It is of interest to note that, for this case, the maximum tensile stress always occurs at the bottom of the bottom layer and is larger in those cases when t_3 is smallest ($t_3 = 1$ in.).

In Fig. (5), $(\sigma_{max})_{tensile}$ is plotted for different layer thicknesses versus the factor R which is defined such that

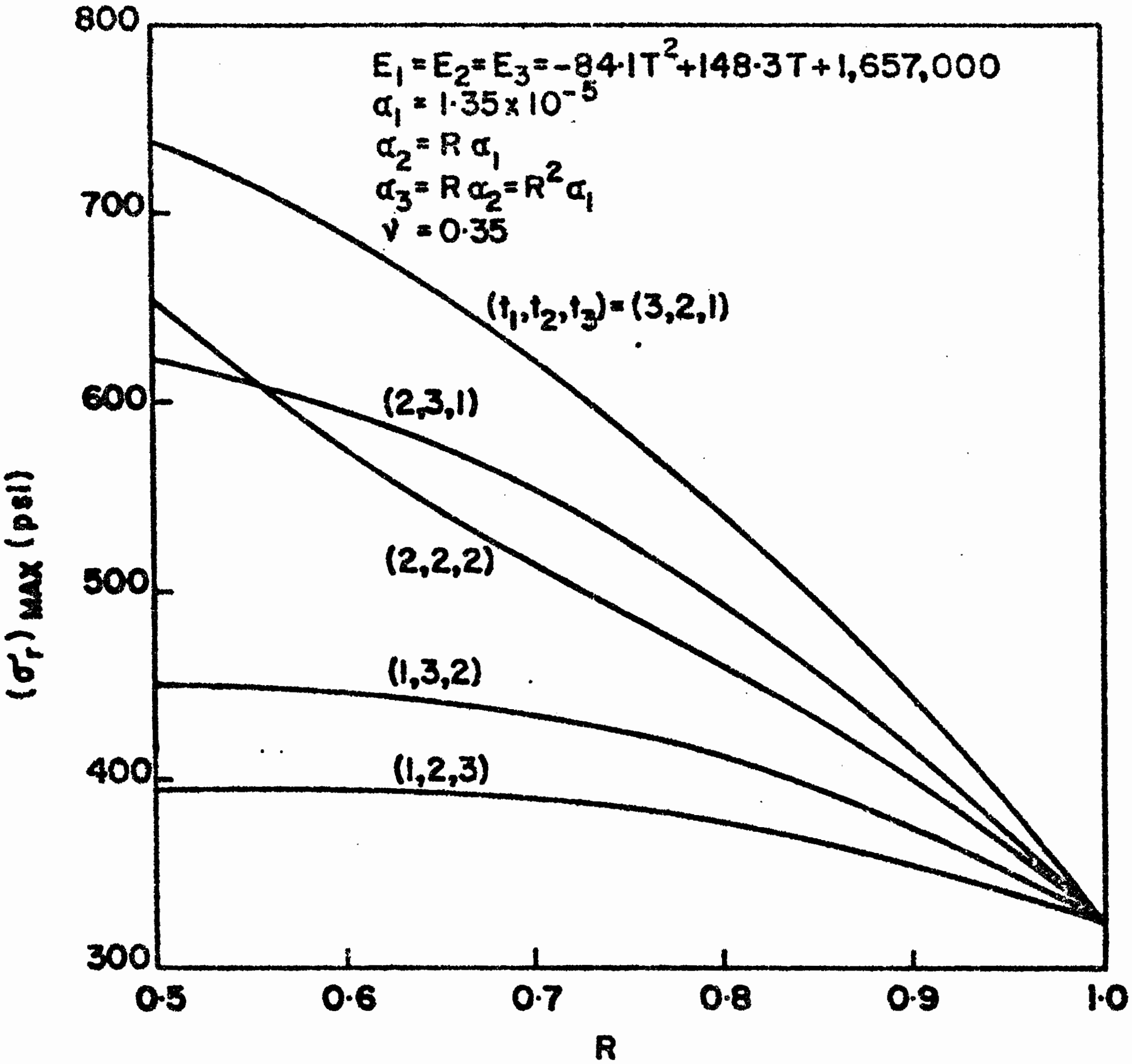

Figure 4 Effect of α_i on $(\sigma_r)_{max}$

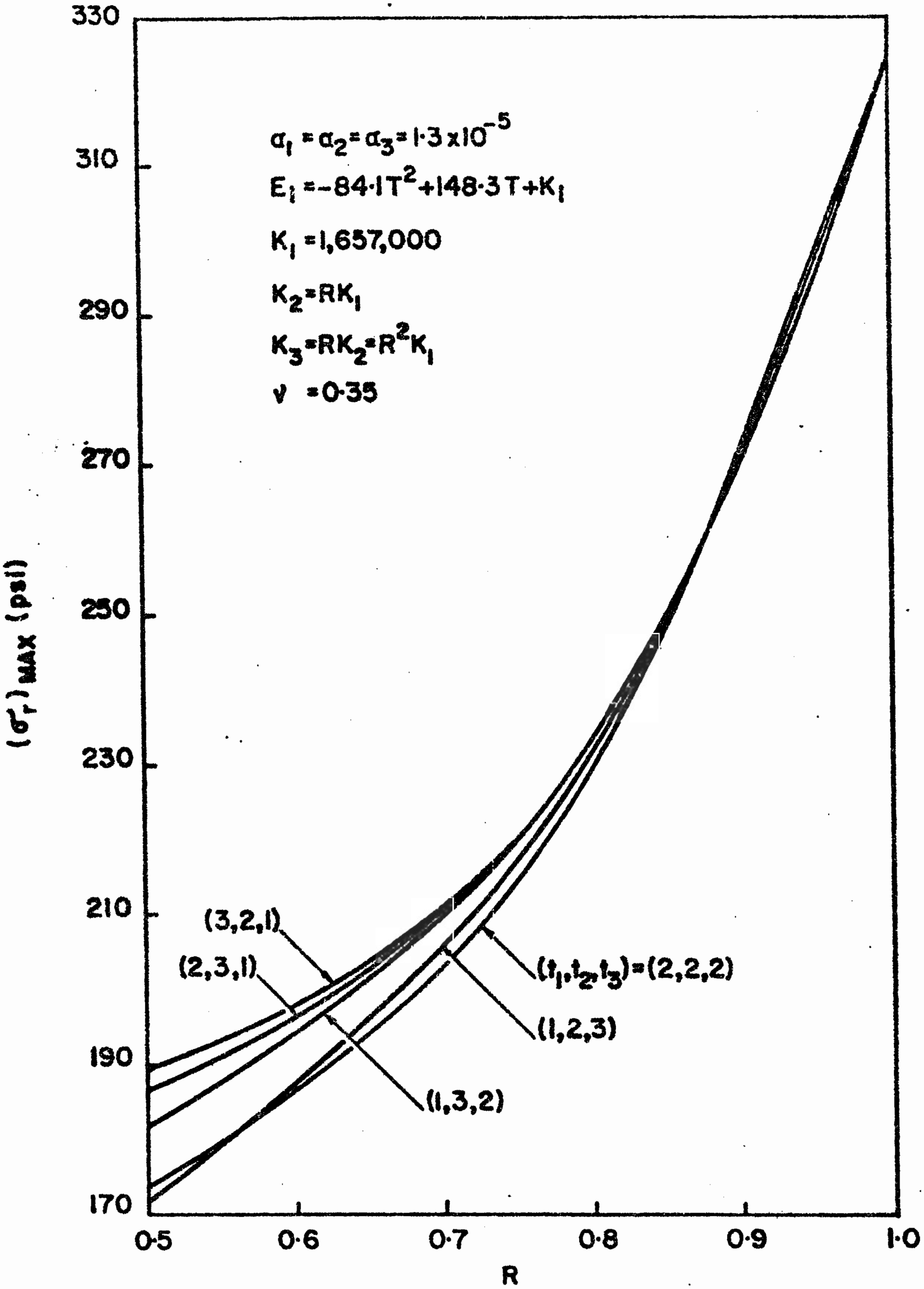

Figure 5 Effect of Young's Modulus on $(\sigma_r)_{max}$

$$E_1 = -84.1\ T\ \ + 148.3\ T + 1{,}657{,}000$$

$$E_2 = -84.1\ T^2 + 148.3\ T + 1{,}657{,}000\ (R)$$

$$E_3 = -84.1\ T^2 + 148.3\ T + 1{,}657{,}000\ (R^2)$$

The coefficients of thermal expansion in the three layers are set equal in Fig. (5). When R = 1, $(\sigma_{max})_{tensile}$ becomes independent of the makeup of (t_1, t_2, t_3) as would be expected. In fact, the maximum stress is quite insensitive to the makeup (t_1, t_2, t_3) for the entire range of R. Notice the striking difference in behavior of the curves of Fig. (5) as compared to those of Fig. (3).

Comparison of the One, Two, and Three Layer Results

One of the goals of this project is to determine if and when the less sophisticated single layer method of determining stress levels can be employed in lieu of the two- or three-layered approach to achieve results of acceptable accuracy. A study of the stress curves presented in the first three reports makes clear the fact that, in general, one would be ill advised to follow such a course. Unless all respective layer parameters (coefficients of thermal expansion, Young's Moduli, Poisson's Ratio) are within 5% of one another, the results of the single layer approximation can vary significantly of those of the more exact analysis.

Work in Progress

The present work is concerned with the solution of an "n" layered plate loaded both thermally and mechanically. The mechanical loading will be assumed to act over a small area of the plate to approximate the effect of wheel load on the system. The interaction of the thermally and mechanically induced effects (stresses) will be investigated.