EFFECT OF STRAIGHTENING DAMAGED STEEL

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Conclusions

While the purpose of this report is to illustrate a way of computing residual stresses, it may be noted that the residual stresses resulting from very large accidental deflections may, in some cases, be less than those resulting from normal procedures. It is hoped that this will be of use to engineers in the application of specifications requirements relative to the straightening of steel sections.

In particular, it is noted that the maximum residual stress (14 ksi) resulting from the straightening of a rather severely but uniformly bent rectangular section (a girder flange) was less than the residual stress (21 ksi) resulting from a routine cambering. Because of the relative magnitude of these residual stresses it appears that the calculation of residual stresses should be considered when uniform (not kinked) accidental bending of structural members occurs. Such calculations would be of assistance to those engineers responsible for making the final decision as to whether accidentally bent steel may be straightened and used or whether it must be replaced.

Summary

The maximum residual stresses are computed for two cases in which steel members were bent by loads which caused yielding of some portion of the cross section.

First, a maximum residual stress of about 21 ksi was found in a W 33 x 200 when cambered to an extent permitted by the AISC Specification. (See Fig. 5 and Section 2·3.)

Second, a maximum residual stress of about 14 ksi was found in a rectangular beam which had been straightened after having been bent to curvature three times the curvature at which yielding first occurred. This case is considerably more severe than might occur in the lateral deflection of a large girder due to a construction accident.

1.0 INTRODUCTION

Of the myriad of mishaps which can occur during any given construction project, one of the most common, and most irritating to deal with, is the accidental damaging of primary structural members. These include all beams, columns and girders which have been specifically designed and fabricated for a particular job. The cost in both time lost in waiting for replacements and money spent in purchasing these replacements can often be very great. It is in the best interest of both the contractor and the customer to know when (or if) a given member can be straightened after being accidentally bent, or when enough damage has been done to the member to warrant its replacement. Most construction codes are non-specific in the establishment of a level of damage which necessitates replacement of a given member. Typical examples are contained in recent editions of the "Connecticut Specifications for Roads, Bridges, and Incidental Construction -- Form 811: Sect. 6.03.03-49" and "AASTO Standard Specifications for Highway Bridges: Sect. 2.10.57."

The field application of those sections can, in some cases, be difficult and open to considerable difference in interpretation. Cases where "fracture" occurs cause no great difficulty and either condemnation or suitable repair can be undertaken. However, in cases where straightening is possible without fracture, the assessment and interpretation of the words "other injury" can easily result in divergent opinion. The primary concern, in such cases, is whether the structure or structural component in its straightened state is in compliance with the intent of the design.

One of the crucial points in the application of the above mentioned specifications was the silence, of these and of all codes, with regard to the effect of such straightening on ultimate strength and with regard to the intensity of accidental deformation which is correctible. With respect to both of these important points the latest edition of the A.I.S.C. Manual of Steel Construction 7th Edition has printed timely and significant comments; the following (underlining added) is reproduced from page 6-11.

USE OF HEAT TO STRAIGHTEN, CAMBER, OR CURVE MEMBERS

"With modern fabrication techniques, a controlled application of heat can be effectively used to either straighten or to intentionally curve structural members. By this process, the member is rapidly heated in selected areas; the heated areas tend to expand but are restrained by adjacent cooler areas. This action causes a permanent plastic deformation or "upset" of the heated areas, and, thus, a change of shape is developed in the cooled member.

"Heat straightening" is used by both normal shop fabrication operations and in the field to remove relatively severe accidental bends in members. Conversely, "heat cambering" and "heat curving" of either rolled beams or welded girders are examples of the use of heat to affect a desired curvature.

As with many other fabrication operations, the use of heat to straighten or curve will cause residual stresses in the member as a result of plastic deformations. These stresses are similar to those that develop in rolled structural shapes as they cool from the rolling temperature; in this case, the stresses arise because all parts of the shape do not cool at the same rate. In like manner, welded girders develop residual stresses from the localized heat of welding.

In general, the residual stresses from heating operations do not affect the ultimate strength of practical members. Any reduction in column strength due to residual stresses is incorporated in the present design provisions.

The mechanical properties of steels are largely unaffected by heating operations, provided that the maximum temperature does not exceed 1100° for quenched and tempered alloy steels, and 1200° F for other steels. The temperature should be carefully checked by temperature-indicating crayons or other suitable means during the heating process."

The underlined portion of the above excerpt represents a much more clear and forceful statement of these facts than existed in the 1963 edition of the A.I.S.C. Manual. Indeed, one of the two main purposes of this research project was to assess the degree of residual stresses induced by the bending and subsequent straightening of steel members. While the methods of determining such residual stresses are quite straightforward, it appears to be a topic which is omitted from familiar texts and references: this topic forms the body of this report. In addition, a M. S. thesis[1] was written by Michael C. Apostal which considered the way in which the Bauschinger Effect influences the strength of straightened members.

[1] Apostal, M.C.; M.S. Thesis, University of Connecticut, 1974. The Bauschinger Effect and Its Implications in the Straightening of Accidentally Bent Structural Steel Members.

2.0 RESIDUAL STRESSES

Residual stresses are present in rolled structural shapes as a result of non-uniform cooling which takes place after rolling is completed and the section is allowed to attain room temperature. These stresses are not small and at places may approach the yield stress of the material; they may be removed by the process of annealing; but, this is seldom done for steel used in structural applications.

Residual stresses are also introduced into a steel section wherever, and for whatever the reason, the section is deformed in such a way that a portion of the material forming the section is strained beyond the yield point. In some cases, such deformation will be occurring accidentally - in other cases, it will be purposely introduced; for example, when a rolled section is cambered.

In order to provide an illustration of the order of magnitude of the residual stresses which can be induced in steel sections that are purposely "bent", we shall determine the residual stresses in a W 33 x 200 section due to cambering.

2.1 Radius of Curvature Due to Cambering

For a W 33 x 200 beam of length 30 ft, an allowable magnitude of camber is 2 inches.* For this camber, the corresponding final radius of curvature of the beam is 675 ft or 8,100 inches. See Figure 1.

In the determination of the radius of curvature, an arc of a circle has been used; this corresponds to the section being bent by equal and opposite couples, while the actual cambering might be done by a series of jacks exerting concentrated forces. Since in a circular arc, the radius of curvature is constant, we obtain the largest camber for a given minimum radius of curvature. Any curve other than a circular arc would result in more severe bending for a given amount of camber.

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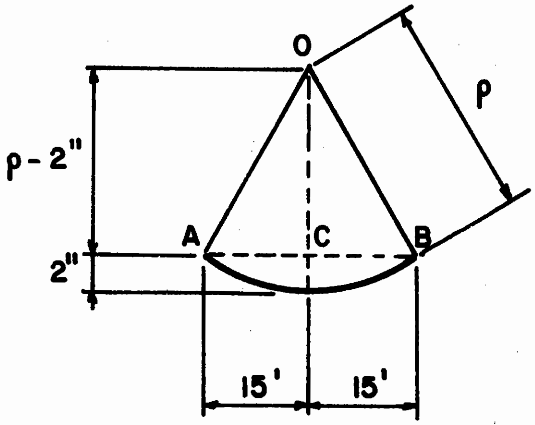
* P 1-125 A.I.S.C. Manual 1970

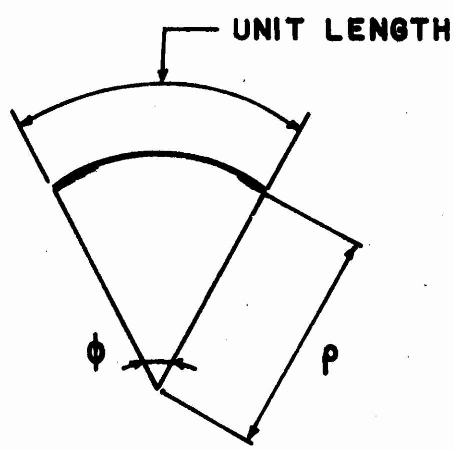


IN TRIANGLE OCB:

$$\rho^2 = (\rho - 2")^2 + (15 \times 12")^2$$

$$\rho = 8101" \approx 8100"$$

FIG. 1



$$\phi = \frac{1}{\rho}$$

FIG. 2

For convenience, the curvature ϕ of the beam may be defined as the reciprocal of the radius of curvature. In addition, we note that the angle ϕ subtends a unit length of the bent beam. See Fig. 2.

2.2 STRESS-STRAIN RELATIONSHIP

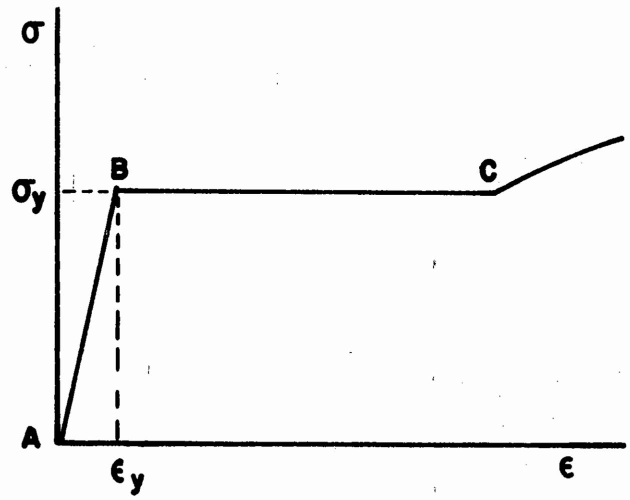
It is assumed that the stress-strain relationship is the same for tension and for compression and is as shown in Figure 3. This consists of an elastic range, AB, in which stresses are less than the yield stress σ_y and the strains are less than the strain at initial yielding ϵ_y. After yielding occurs there exists a plastic range, BC, in which the yield stress σ_y is sustained for strains many times larger than ϵ_y. Finally, there exists a range of strain hardening associated with very large strains. In this report we shall be concerned with deformations which, while large, do not involve unit strains in the strain hardening range.

As an axially loaded specimen is unloaded it behaves "elastically" even if it has been strained into the plastic range. That is, a specimen loaded to point D of Figure 4 when unloaded will proceed along the dashed line DE (the strain AE would be the permanent set caused by the loading). Subsequent reloading will precede along the path EDC if it is in tension or along the path EFG if it is in compression. In all cases the maximum stress is assumed to be σ_y and the slope of all "elastic" portions is assumed to be E = 30,000 ksi.

The steel used in the following examples will be A36 and its properties will be taken as shown in Figure 3, i.e., σ_y = 36 ksi and ϵ_y = 0.0012 in/in.

2.3 RESIDUAL STRESSES DUE TO CAMBERING

In this section we shall compute the residual stresses induced in a W 33 x 200 beam of length 30 feet if cambered to the extent permitted by standard practice (radius of curvature = 8,100 in. see Section 2.1). The details of the computation are given in Figure 5 and may briefly be described as follows:



FOR A 36 STEEL:

σ_y = 36 KSI

ϵ_y = .0012

FIG. 3

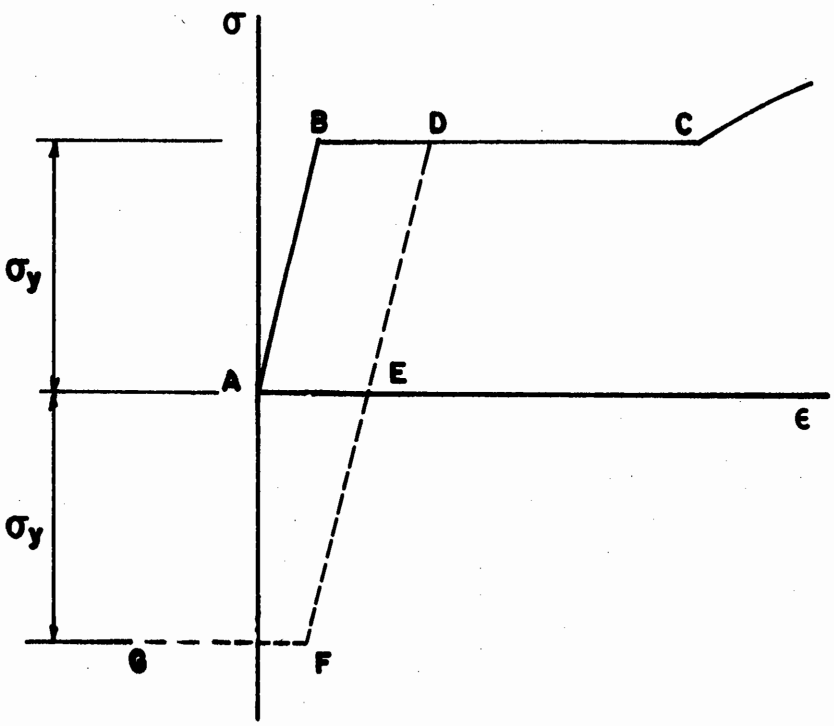
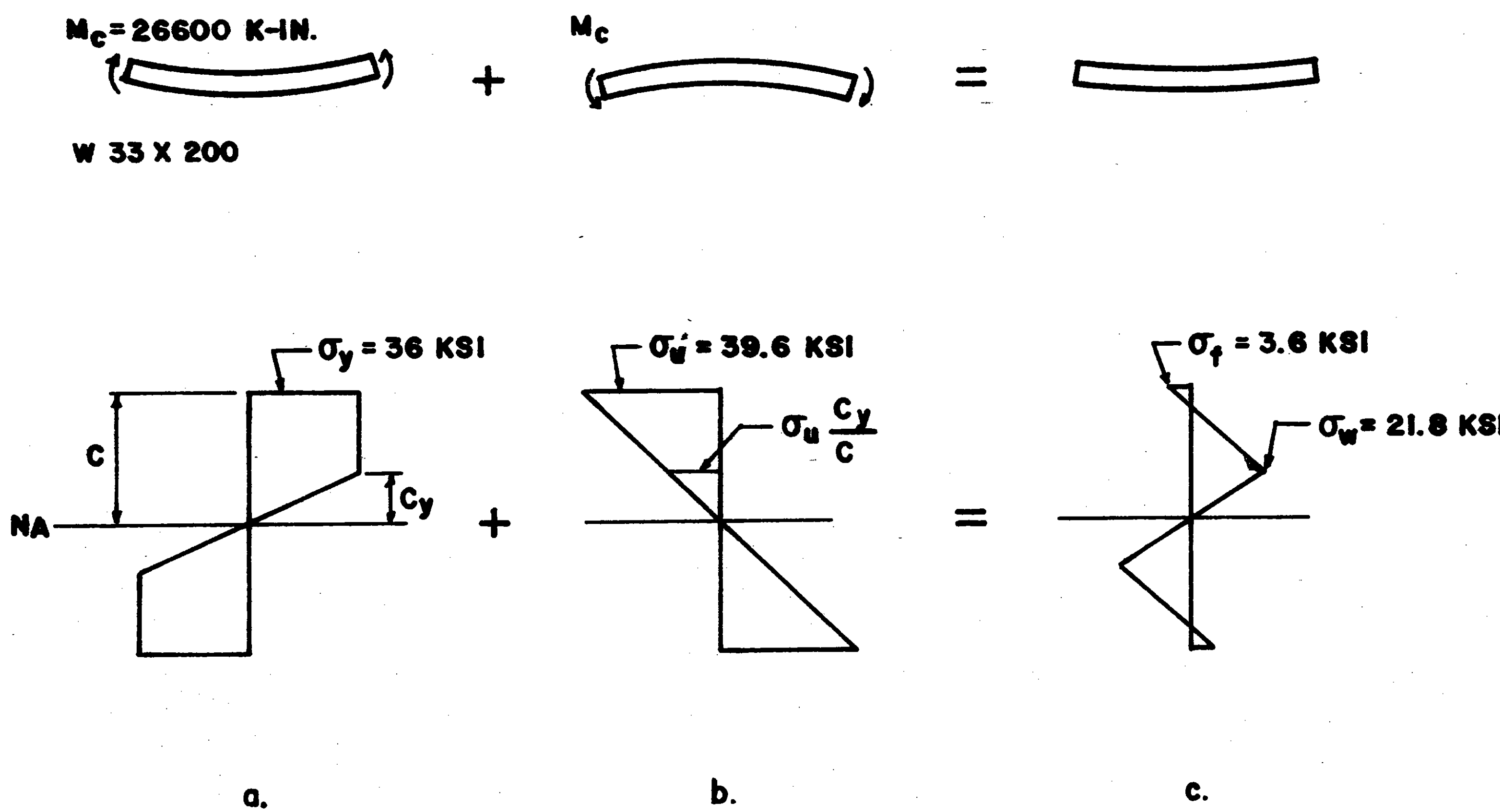


FIG. 4



Mc= 26600 K-IN.
W 33 X 200
Mc
NA
C
Cy
σy = 36 KSI
σu' = 39.6 KSI
σu Cy/C
σf = 3.6 KSI
σw = 21.8 KSI
a.
b.
c.

FIG. 5

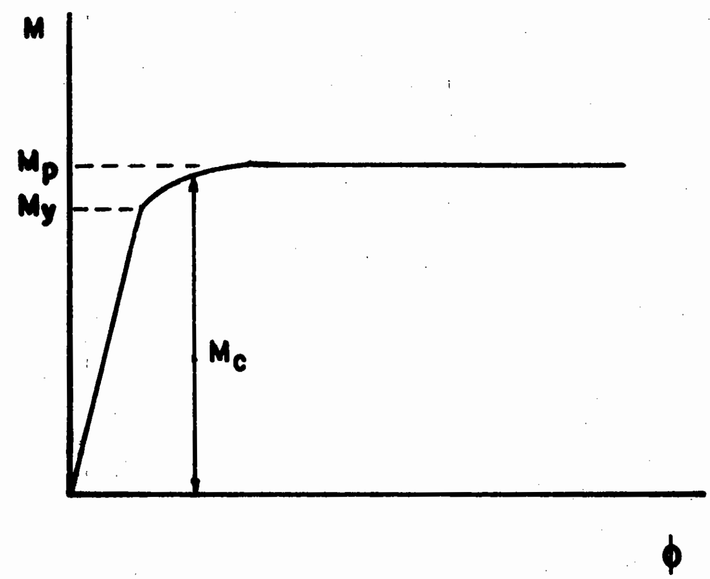
a. Loading Phase. A short length of the beam is bent, by equal and opposite external couples of magnitude M_c. The magnitude of M_c must be larger than the yield moment M_y, yet smaller than the plastic moment M_p, i.e., $M_y<M_c<M_p$. The loading will cause yielding first in the extreme fibers and then in a zone extending from the extreme fibers to points located at a distance c_y from the neutral axis. The central portion of the beam will remain elastic with stresses ranging from zero to σ_y. It may be noted that the curvature with M_c applied is greater than the desired final curvature. The magnitude of the applied couple M_c may be established by considering the M-ϕ curve for a typical W-shape beam; see Figure 6. The value $M_c \simeq 1.10\ M_y$ is chosen after noting that the curve becomes asymptotic to the value $M = M_p$ and that the strain involved is several times greater than the strain associated with the inception of yielding.

The stress distribution of the beam at the end of this loading phase is shown in the first portion of the "picture equation" given in Figure 5. At this point the distance c_y is still unknown, it is the distance from the neutral axis to the point in the cross section of the beam to which yielding has penetrated.

b. Unloading Phase. The final cambered beam is, of course, subjected to no external loading, thus the effect of the applied couples M_c must be removed. This may be done by applying to the beam, couples of magnitude M_c directed opposite to those used to originally load the beam. The net external loading of the beam is then zero. During this "unloading" we note that the beam behaves elastically. Figure 5. This is due to the fact that each fiber is being unloaded and all stresses will be less than σ_y.

The maximum stress (or more properly the maximum stress change) caused by this "elastic" unloading is

$$\sigma_u = M_c/S = (26{,}600\ \text{k-in.})/(671\ \text{in.}^3) = 39.6\ \text{ksi}$$



$$M_p = \frac{Z}{S} M_y \approx 1.13\, M_y$$

$$M_c \approx 1.10\, M_y$$

FOR W33 x 200

$C = 16.5$ IN.

$S = 671$ IN.^{3} $\quad \sigma_y = 36$ KSI

$Z = 756$ IN.^{3}

$M_y = S\,\sigma_y = (671)(36) = 24156$ K-IN.

$M_p = Z\,\sigma_y = (756)(36) = 27216$ K-IN.

$M_c \approx 1.10\, M_y = 26572$ K-IN.

FIG. 6

The stress distribution due to this unloading phase is shown in the second portion of the "picture equation" of Figure 5.

c. Final Cambered Beam. The stress distribution of the final cambered beam is shown in the last portion of the "picture equation" of Figure 5. We note that the external loading is zero and that the final stresses are obtained by adding the stresses shown in the first two portions of the figure.

We must now relate the stresses found above to the strains and curvatures of the beam. We recall from Figure 2 that $\phi = 1/\rho$ and that for an elastic portion of a beam

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{\sigma I}{y} \cdot \frac{1}{EI} = \frac{\sigma}{yE}$$

we observe that the central portion of the cambered beam, $y \leq c_y$, has remained elastic. Thus, the above equation applies to the central portion of the beam; in particular, it applies to the boundary of the central portion where $y = c_y$. At that boundary, see the last portion of Figure 5, we have

$$\frac{1}{\rho} = \frac{\sigma_w}{c_y E} \qquad \text{or} \qquad \sigma_w = \frac{c_y E}{\rho}$$

or, since $\rho = 8100$ in, $E = 30{,}000$ ksi and $\sigma_y = 36$ ksi, we write

$$\sigma_w = \frac{c_y\,(30 \times 10^3)}{8100} = 3.70\ c_y$$

On the other hand, by adding the stresses at the same point from the first two portions of the "picture equation" of Figure 5, we obtain

$$\sigma_w = \sigma_y - \sigma_u \frac{c_y}{c} = 36 - 39.6\,\frac{c_y}{16.5} = 36 - 2.4\ c_y$$

Equating the above expressions for σ_w, we write

$$3.70\ c_y = 36 - 2.4\ c_y$$

$$6.1\ c_y = 36$$

$$c_y = 5.9 \text{ in.}$$

This distance is, for a final radius of curvature of 8100 in, the distance from the neutral axis to the edge of the core of the beam which remained elastic throughout the cambering.

We now compute the final value of σ_w, namely

$$\sigma_w = 36 - 2.4\, c_y$$

$$= 36 - 2.4\ (5.9)$$

$$= 21.8 \text{ ksi}$$

The final stress in the outside fibers of the flange (see Figure 5) is

$$\sigma_y = 36 - 39.6 = -3.6 \text{ ksi}$$

It may be noted that in the final cambered beam the residual stress in the extreme fiber of the flange on the convex (stretched) side is compression while the residual stress in the flange on the concave side is tension.

The maximum residual unit strain is, of course, that strain associated with the final curvature of 8100 in. It occurs at the extreme fibers in the flange and is

$$\varepsilon = \frac{1}{\rho}\, c = \frac{16.5 \text{ in.}}{8100 \text{ in.}} = 0.00204 \text{ in/in.}$$

3.0 TYPICAL RESIDUAL STRESSES DUE TO ACCIDENTAL BENDING

The accidental bending of beams or of built-up girders often involves the lateral bending of the top and/or bottom flanges. The prime reason for this is that in its final design position, the lateral bracing provided for a large built-up girder is quite adequate to prevent any lateral buckling. However, during erection while the girder is still a separate unit, lateral buckling may become a reality. For example, if a long girder, normally picked up at about the outside fifth points of its span, see Figure 7, accidentally touched an abutment at one of its ends, the effective length would be greatly increased and in many cases, lateral buckling would occur. Such lateral buckling would result in the girder being bent about its minor axis; in such a case, the flanges would behave as rectangular beams of "height" b and "width" t (Figure 8). It is for this reason that we shall, in the following sections, consider the bending and straightening of long rectangular beams.

In order to focus on cases of practical value which might occur in practice, we shall consider a 140-ft welded plate girder with 18 in. x 1 in. flanges. In such a girder an accidental lateral buckling, as described above, could cause temporary lateral deflections of the order of magnitude of six feet. While quite large by normal structural standards such deflections do not necessarily indicate that the girder cannot, or should not, be straightened and used for its intended purpose. Indeed, the deflection due to elastic deformation alone is about 3.3 feet, 39.2 in. to be precise, see Figure 8. Thus even for a "six-foot" deflection only about 2.7 feet is due to plastic deformation. Naturally, if all of the plastic deformation is the result of a single kink or sharp bend, localized material damage would be expected.

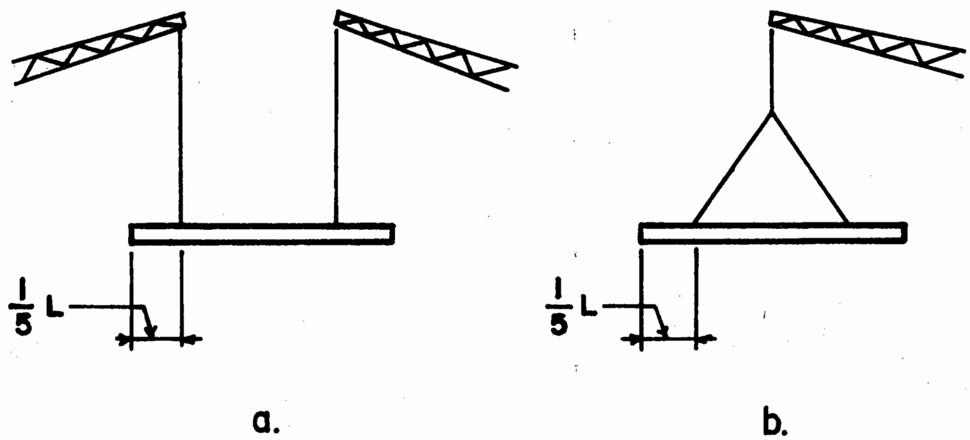


FIG. 7

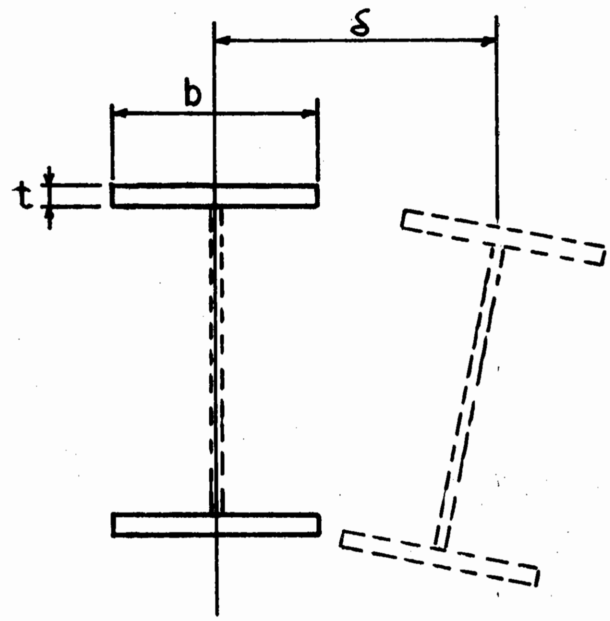


FIG. 8

However, visual inspection (coupled if necessary with tape and transit measurements) can easily show whether the deformed member has been kinked or whether the deformation is distributed over large portions of the beam. If the latter is the case, then even a "large" deflection may result in rather small permanent unit strains. In the case referred to above which had a "six-foot" deflection the corresponding maximum unit strains are of the magnitude 0.002 in/in. or only about twice the yield strain of A 36 steel (ε_y = 0.0012 in/in.).

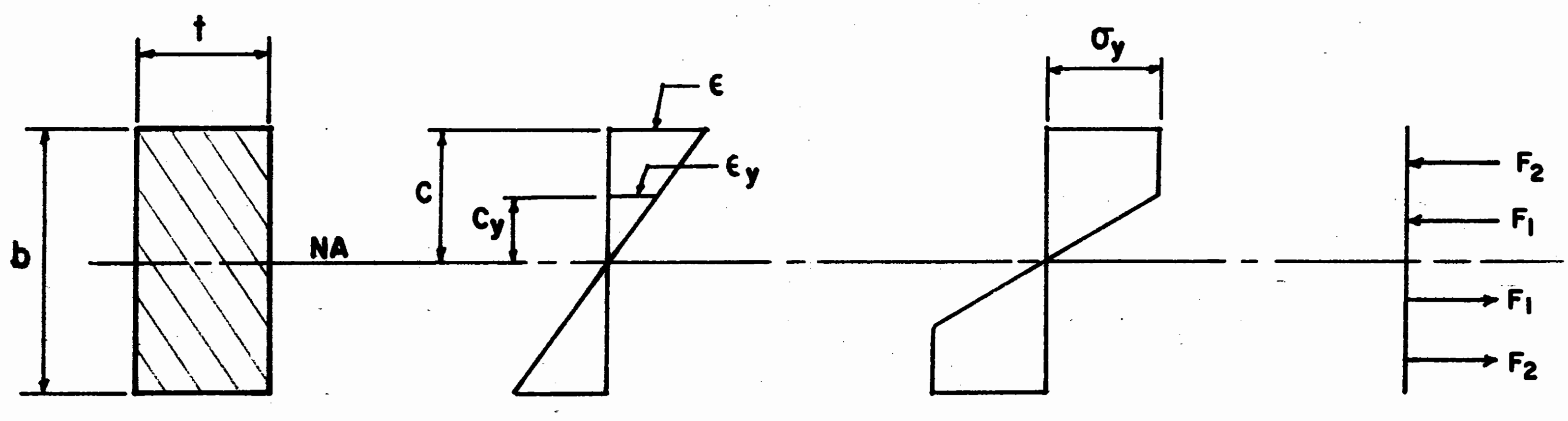
4.0 BENDING OF RECTANGULAR SECTIONS

We shall now consider the bending of a steel beam of rectangular cross section by a couple of magnitude M_1. Grade A-36 steel is assumed; the stress-strain relationship of this steel has been given in Fig. 3. The strain distribution, which is as always in elementary analyses assumed to be linear, is shown in Fig. 9b. The corresponding stress distribution is shown in Fig. 9c. It is noted that the boundary between elastic and plastic deformation occurs at a distance c_y from the neutral axis. Replacing the distribution of unit stress of Fig. 9c by equivalent concentrated forces (Fig. 9d), we may sum moments about the neutral axis and obtain the following, well-known, expression for the applied moment.

$$M_1 = \sigma_y \, t \, \frac{3\,c^2 - c_y^{\,2}}{3} \qquad (1)$$

If $c_y = c$, we obtain the case when yielding first occurs for which $M_1 = M_y$, that is

$$M_1 = M_y = \frac{2}{3}\,\sigma_y \, t \, c^2$$



RECTANGULAR CROSS SECTION
a.

STRAIN DISTRIBUTION
b.

STRESS DISTRIBUTION
c.

FORCES
d.

FIG. 9

Since this latter case consists of entirely elastic deformation we may also use the elementary formula $M = \sigma I/c$ and obtain

$$I = \frac{tb^3}{12} = \frac{t(2c)^3}{12} = \frac{2}{3}\, t\, c^3$$

$$M_y = \frac{2}{3}\,\frac{\sigma_y\, tc^3}{c} = \frac{2}{3}\,\sigma_y\, tc^2 \tag{2}$$

Combining equations (1) and (2) we obtain after rearrangement, for $M > M_y$,

$$M_1 = M_y\left[\frac{3}{2} - \frac{1}{2}\,\frac{(c_y)^2}{c^2}\right] \tag{3}$$

The beam may be "unloaded", see Sect. 2.3, by considering the addition of couples equal and opposite to the couples, of magnitude M_1, which were applied to cause the original yielding of the beam. As previously noted this unloading action is completely elastic. The final stress distribution is shown in Fig. 10c. In addition, we may determine the curvature of the beam due to the applied couple M_1 (from Figure 10a) and in its final deformed state (from Fig. 10c). The curvature under the applied couple M_1 is

$$\phi_1 = \frac{\sigma_y/E}{c_y}$$

For the unloading case, which is "elastic", we may write

$$\phi_2 = \frac{1}{\rho_2} = \frac{M_2}{M_y}\,\frac{M_y}{EI} = \frac{M_2}{M_y}\,\frac{1}{\rho_y} = \frac{M_2}{M_y}\,\phi_y$$

where ϕ_y is the curvature at the onset of yielding, i.e., $\sigma_{max} = \sigma_y$,

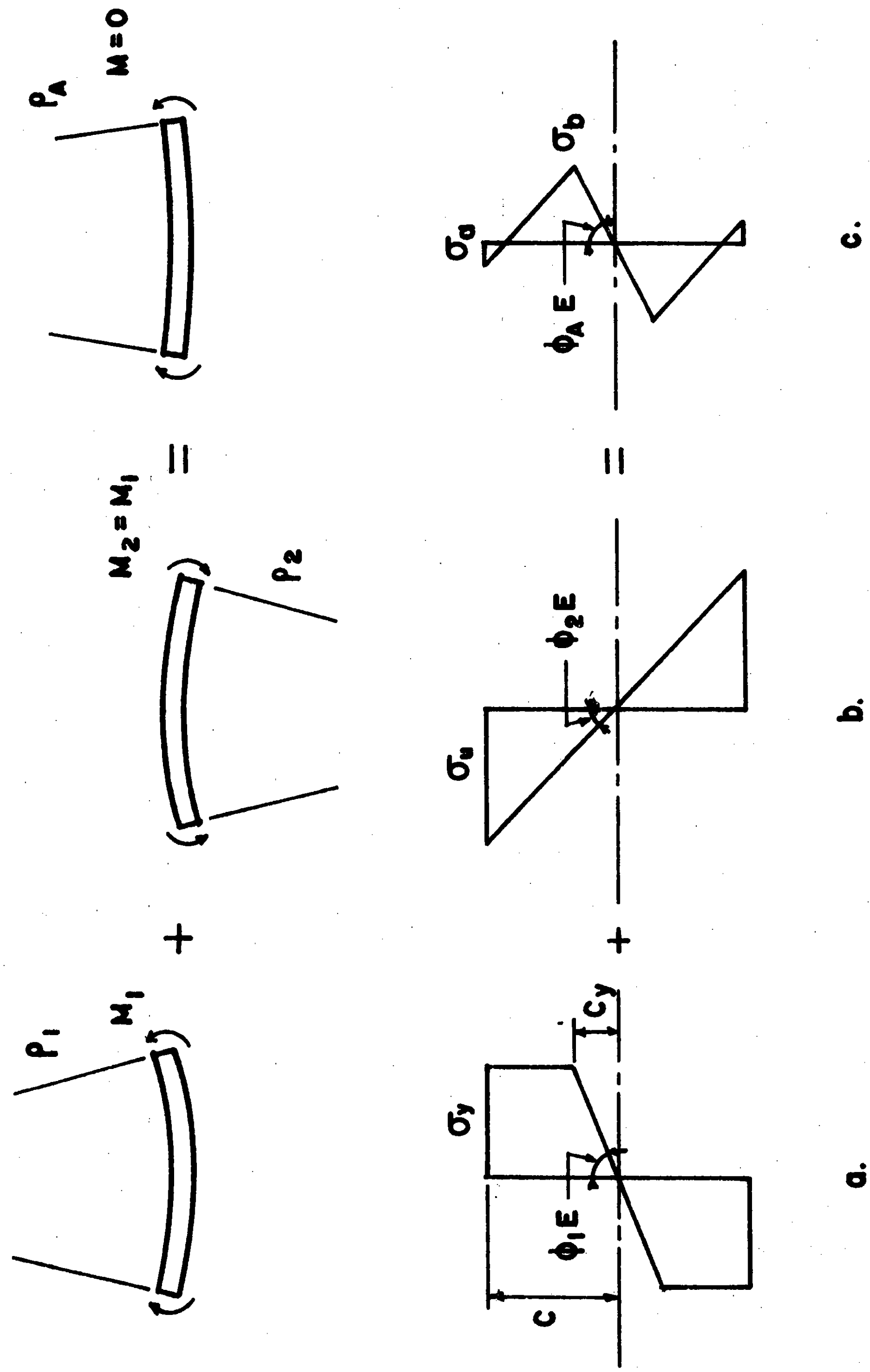


FIG. 10

$c_y = c$, and $M = M_y$. From Fig. 10 we note that

$$\phi_A = \phi_1 - \phi_2 = \phi_1 - \frac{M_1}{M_y} \phi_y$$

Using the above formulas, together with Figures 9 and 10 we may construct Table 1 which gives the stresses and strains at any desired degree of plastic deformation.

4.1 STRAIGHTENING OF A DEFORMED BEAM OF RECTANGULAR CROSS SECTIONS

In Section 4.0 an initially straight rectangular beam was bent by couples of sufficient moment ($M > M_y$) to cause plastic deformation. These couples were then removed and the permanent deformation (curvature ϕ_A) and the residual stresses σ_a and σ_b were determined. In this section we shall consider the process of ~~and results of~~ returning the beam to its original straight condition. While the curvature of the beam will again be zero, we will find that the stresses are not zero and that residual stresses remain in the straightened beam. The method of analysis used consists of two phases, namely;

A. Reverse Loading Phase. As a result of the bending mentioned above the beam has a curvature ϕ_A, assumed to be concave up, and residual stresses σ_a and σ_b (see Fig. 11a). We shall now apply equal and opposite couples M_3 as shown in Fig. 11b. The stresses caused by M_3 cannot be determined merely from Fig. 11b since they are being added to those of Fig. 11a and the total stress in no case can exceed σ_y. Thus the total stresses consist of the algebraic addition of the stresses in parts a and b of Fig. 11 with the restriction that $\sigma_{max} = \sigma_y$; the corresponding radii of curvatures, ρ_A, ρ_3, and ρ_B are also shown in Fig. 11.

B. Unloading of Reverse Loading Phase. The reverse loading couples M_3 produced the stresses and curvature of Fig. 11c which for convenience are repeated as Fig. 12a. To this loading, we add the loading shown in

Table 1

$\frac{c_y}{c}$	$\frac{c}{c_y}$	Initial ε_{max} in./in.	M_1 Kip·in.	σ_a	σ_b	ϕ_A
1.0	1.0	0.0012	1944	0	0	0
0.8	1.25	0.0015	2294	6.5	2.0	0.00001
0.667	1.50	0.0018	2484	10.1	5.3	0.00003
0.5	2.0	0.0024	2673	13.6	11.25	0.00009
0.333	3.0	0.0036	2807	16.0	18.67	0.00021
0.25	4.0	0.0048	2855	16.8	22.8	0.00038
0.20	5.0	0.006	2877	17.3	25.3	0.00047
0	∞	∞	2916	18.0	36.0	∞

Values in this table are for a 1 in. x 18 in. rectangular beam (see Fig. 13) with σ_y = 36 ksi.

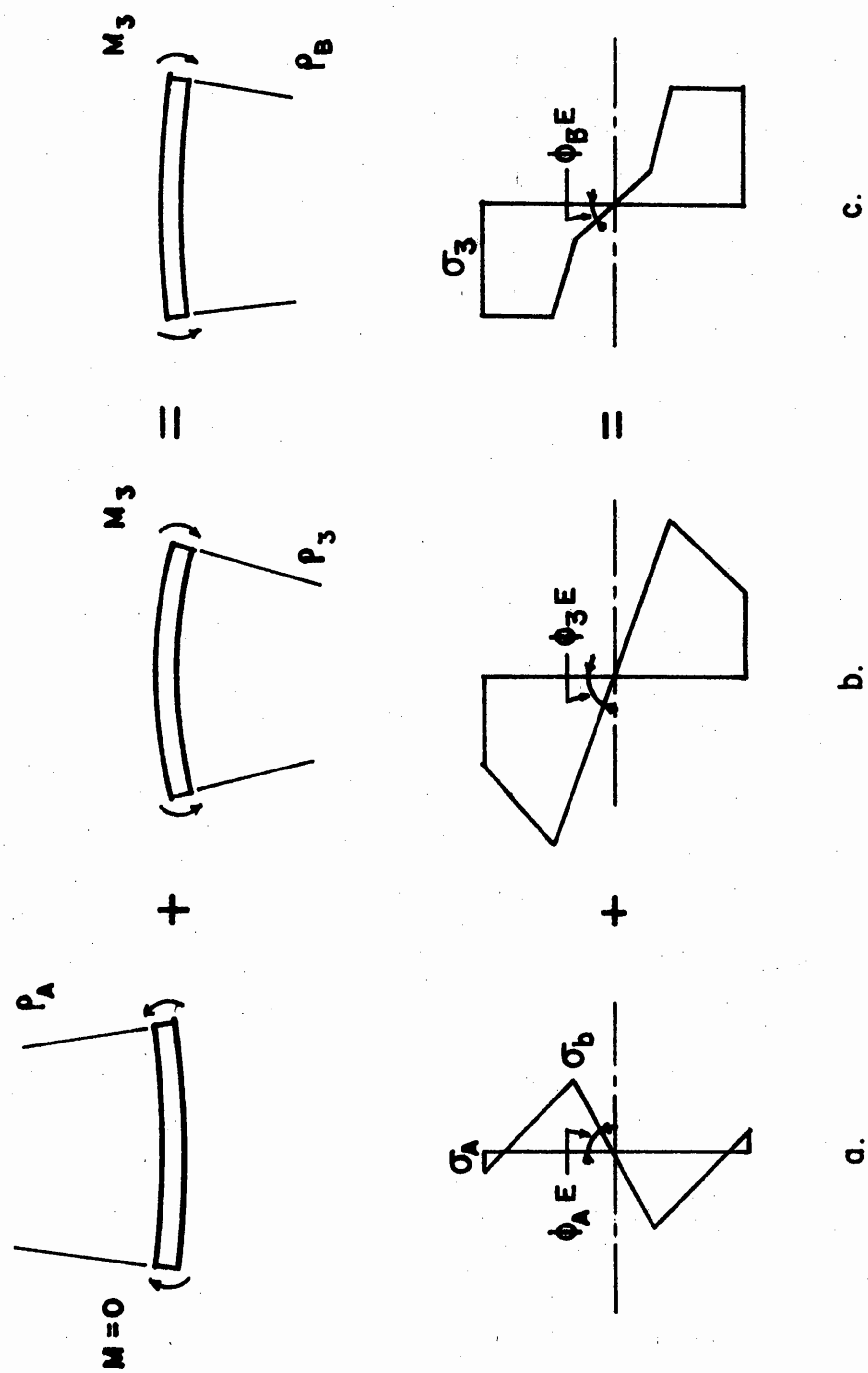


FIG. 11

Fig. 12b, which consists of couples of magnitude M_4 which are equal in magnitude and directed opposite to couples M_3. The result is the unloaded beam of Fig. 12c. If the magnitude of the couples M_3 have been correctly chosen, the final curvature of the beam $\phi_B - \phi_4$ will be zero and we will have obtained the straight beam drawn in Fig. 12c.

The determination of the required magnitude of M_3 may be done analytically, with or without an associated computer program. It also may be done by a semi-graphical method; and since the semi-graphic method also shows more clearly the physical steps involved, we shall outline it in the following section.

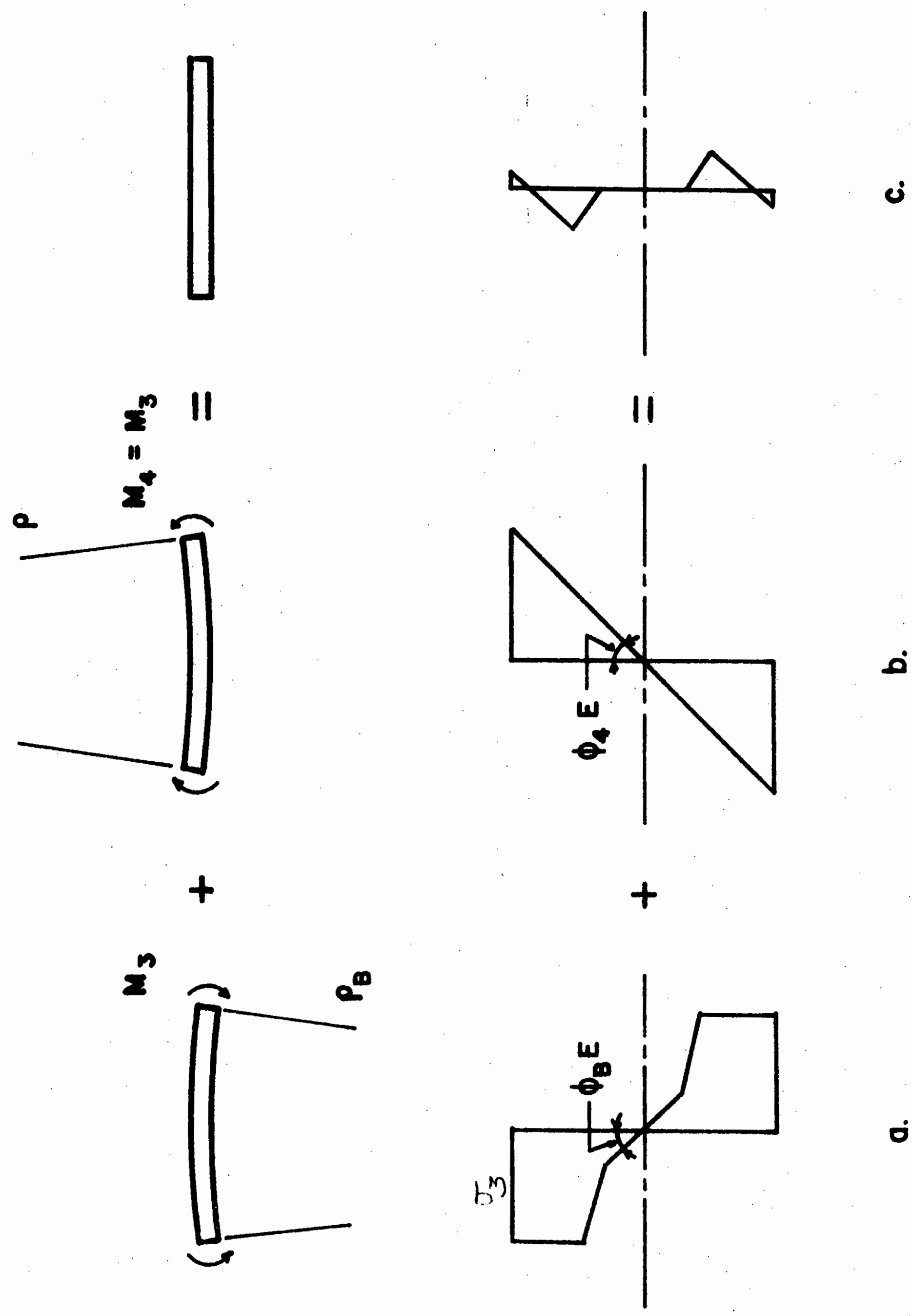


FIG. 12

4.2 SEMI-GRAPHICAL METHOD TO DETERMINE M_3

It is desirable at this point to focus on a numerical example; we shall choose the beam and loading shown in Fig. 13, and consider a case in which the maximum unit strain due to bending was three times the unit strain at the yield point of the steel (the maximum curvature was, of course, also three times the corresponding curvature at yielding).

Referring to Table I we find the following data for this case:

$c/c_y = 3$	$\varepsilon_{max} = 0.0036$
$\sigma_y = 36$ ksi	$M_y = 1944$ k-in.
$M_1 = 2807$ k-in.	
$\phi_A = 0.00021$	$\rho_A = 4762$ in.
$\sigma_a = 16.0$ ksi	$\sigma_b = 18.67$ ksi

The method used to determine, semi-graphically, the final residual stresses in the straightened beam is as follows.

On a single diagram, Fig. 14, we shall draw* the following stress diagrams:

a. The residual stresses which resulted from the original loading and subsequent unloading of the beam (o, m, n); see Fig. 11a or Fig. 10c.

b. The negative of the stresses which resulted from the reversed loading phase (o, q, p); these are caused by the couple M_3. See Fig. 11b. Note the figure is as yet incomplete since c_1 is not known.

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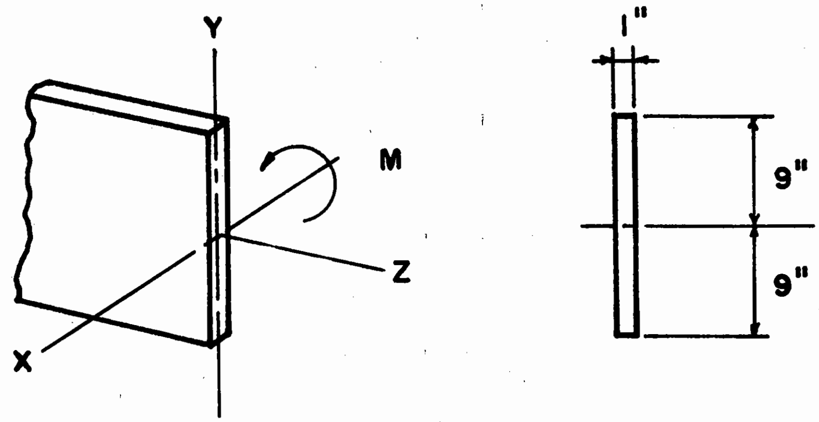
* Because of symmetry only the top half of each stress diagram need be drawn.



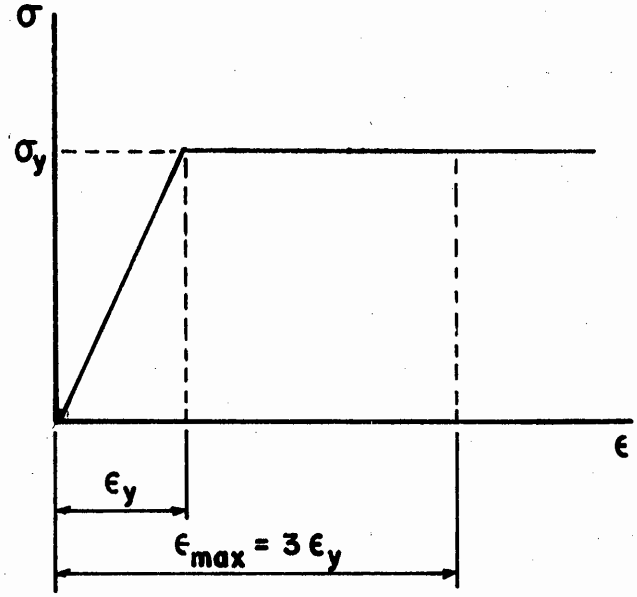


FIG. 13

The negative of the stresses due to M_3 have been drawn so that the addition of the stresses of Figs. 11a and 11b, will be represented in Fig. 14 by the difference between the two stress diagrams -- this difference has been shaded in Fig. 14b. We note that this shaded diagram is identical to Fig. 11c. Since σ_y = 36 ksi is the maximum possible stress, the two sloping portions of the figure, mn and qp, are separated by a horizontal distance which is (to scale) 36 ksi. Indeed, this fact provides a convenient fashion in which we may draw Fig. 14a; namely draw Fig. 11a, then draw a line parallel to and located a horizontal distance σ_y from the stress diagram of the yielded portion of the beam; the diagram is completed by drawing the straight line oq. It is again noted that the location of point q is, at this time, not established; it depends on the yet to be determined values of M_3 and c_1. These values are determined by trial.

For example, if we assume c_1 is 4.0 in, using Fig. 14 we find:

slope of nm = (16 ksi + 18.67 ksi)/(9 in.- 3 in.)

= 5.78 ksi/in.

Horizontal projection of pq = (5.78)(9-4) = 28.90 ksi.

σ_q = 20 + 28.90 = 48.9 ksi

σ_e = (3 in./c_1) 48.9 ksi - σ_b

= (3/4) 48.9 - 18.67 = 18.0 ksi

These values are shown on Fig. 15.

The moment represented by the stresses of Fig. 14b is, of course, M_3 and may be expressed as $M_3 = \int (\sigma)\, y\, dA$ or since dA = t dy; $M_3 = t \int y\, (\sigma\, dy)$. Since in Fig. 14b the abscissa is σ and the ordinate is y,

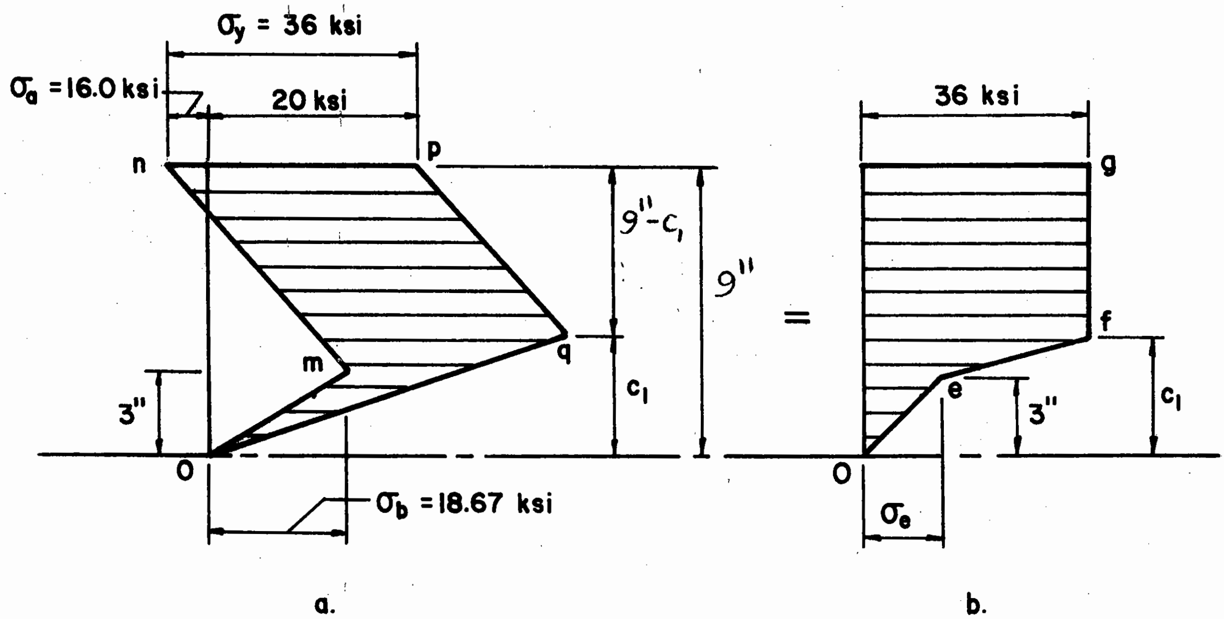
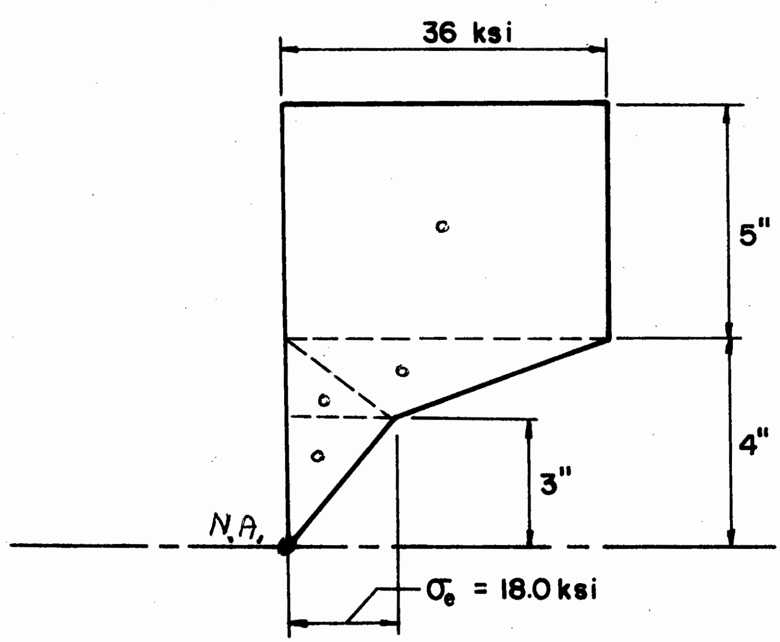


FIG. 14



Values for assumed value c_1 = 4.00"

FIG. 15

we observe that the magnitude of M_3 is proportional to the moment of the area of the stress diagram about the neutral axis.

Continuing the numerical example based on the assumed value c_1 = 4 in., we determine the moment of the area shown in Fig. 15 about the neutral axis. Noting that area is composed of the rectangle and three triangles shown, and recalling that we have drawn only the top half of the stress diagram we write:

$$
\begin{aligned}
(36 \times 5)[9 - 0.5(5)] &= 1170 \\
1/2\ (36 \times 1)[4 - 0.333] &= 66 \\
1/2\ (18 \times 1)[4 - 0.667] &= 30 \\
1/2\ (18 \times 3)(2) &= \underline{54} \\
1/2\ M_3 &= 1320 \\
M_3 &= 2640 \text{ kip. in.}
\end{aligned}
$$

The moment M_3 must now be removed by applying the equal and opposite moment M_4; the stress diagram due to M_4 is elastic, thus triangular in shape, Fig. 16b. But, we also wish the final curvature to be zero, thus the angle ϕ_4 must equal ϕ_B. Since $\phi_B E = \sigma_e/3$ in., and $\phi_4 E = \phi_B E$, we find the maximum stress in Fig. 16b to be $3\sigma_e$. The addition of these two stress diagrams will yield a stress diagram with zero stress in the core of the beam and the residual stresses as shown in the shaded portion of Fig. 16c.

Again continuing the example based on the assumption that c_1 = 4.0 in., we recall that σ_e = 18.0 ksi (see Fig. 15) and find the maximum stress in Fig. 16b to be 3 σ_e = 54 ksi. Since 1/2 M_4 is the moment

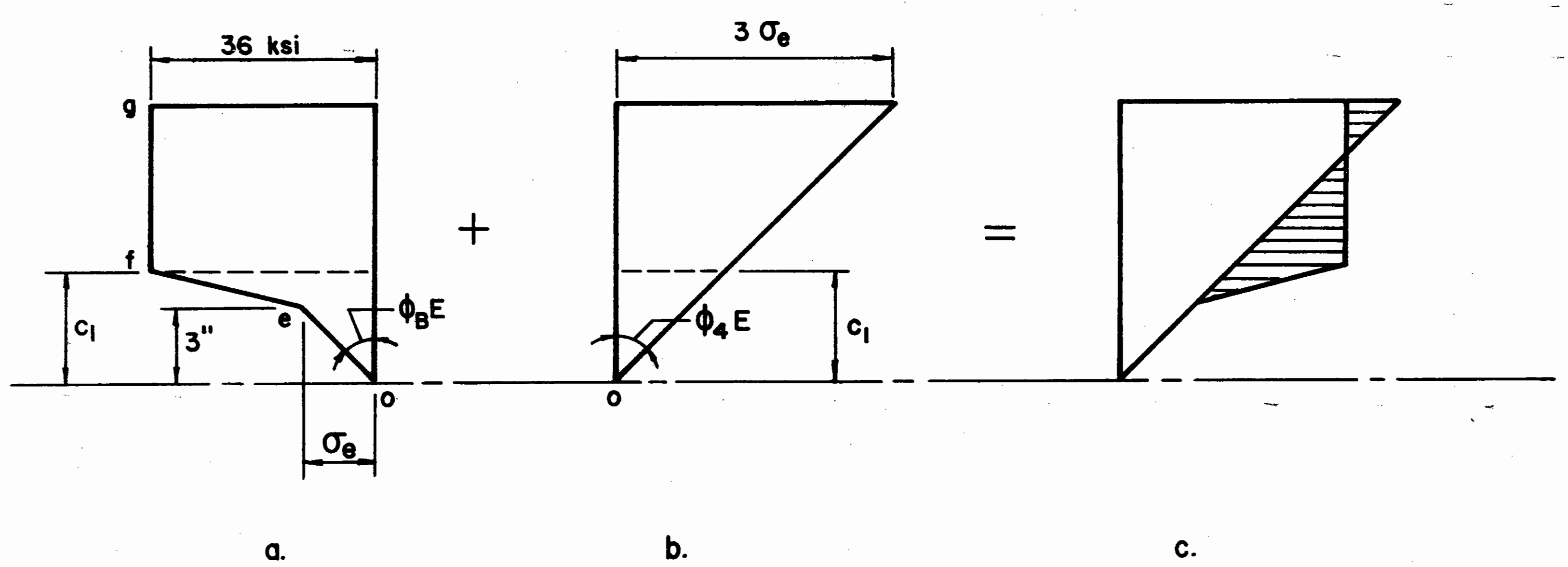


FIG. 16

of the area of Fig. 16b about 0, we have*

$$1/2\ M_4 = 1/2\ (54 \text{ ksi} \times 9 \text{ in.})(6 \text{ in.})$$

$$M_4 = 2916 \text{ kip. - in.}$$

The resulting stress at the outside fiber is 54 ksi - 36 ksi = 18 ksi. At a distance $c_1 = 4.0$ from the neutral axis the resulting stress is

$$\sigma_{c_1} = 36 \text{ ksi} - (3\sigma_e)(c_1/9 \text{ in.})$$

$$= 36 \text{ ksi} - (54 \text{ ksi})(4.0 \text{ in.}/9 \text{ in.}) = 12 \text{ ksi}$$

These stresses are shown in Fig. 17. From a graphical viewpoint these stresses may be obtained by merely extending the line oe through the stress diagram caused by M_3; this is done in Fig. 16c.

It is recalled that we did not know the correct value of M_3 and we originally "guessed" a value. If we had guessed the correct value of M_3, the moment of the "final" residual stresses will be zero about the neutral axis; such is obviously not true in the present case. Indeed, we found M_3 = 2640 kip.-in. and M_4 = 2916 kip.-in.

Thus we must return to Fig. 14a and choose a new trial value of c_1. Repeating the trial process, we will finally determine $c_1 = 4.145$ in. and $M_3 = M_4 = 2610$ kip.-in. This is the final solution, since we have simultaneously arrived at a case where the curvature is zero and where the total resultant of the residual stresses is zero. The residual stresses are shown in Fig. 18. The net external loading is

---

*Since the unloading is "elastic" this can also be obtained by using $\sigma = Mc/I$.

zero. We may note that for the elastic core of the beam ($y \leq y_c$) the final stresses are zero. These fibers never exceeded the yield point and since their strain has returned to zero, their stresses must also return to zero.