THE HYPERLANG PROBABILITY DISTRIBUTION--
A GENERALIZED TRAFFIC HEADWAY MODEL

by

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ABSTRACT

This research is concerned with the development of the hyperlarga probability distribution as a generalized time headway model for single-lane traffic flows on two-lane, two-way roadways. The study methodology involved a process that is best described as "model evolution", and included:

1) Identification of salient headway properties;
2) Construction of mathematical micro-components to simulate essential headway properties;
3) Integration of the micro-components into a general mathematical headway model;
4) Numerical evaluation of model parameters; and
5) Statistical evaluation of the model.

The proposed hyperlarga headway model is a linear combination of a translated exponential function and a translated Erlang function. It is expressed mathematically as

\[
F(h \geq t) = \frac{a_1}{y_1} + \frac{a_2}{y_2} \sum_{x=0}^{\infty} \frac{k(t - \delta_x)}{y_2} \cdot \frac{x}{k(t - \delta_x)}
\]
where:

\[ P(h > t) = \text{the probability that a time headway is equal to or greater than } t; \]
\[ t = \text{any time duration between the limits } \text{Min}(t, b) \text{ and } \infty \]
\[ a_1 = \text{the portion of free headways in the traffic stream;} \]
\[ a_2 = \text{the portion of constrained headways in the traffic stream;} \]
\[ Y_1 = \text{the mean of the free headway distribution;} \]
\[ Y_2 = \text{the mean of the constrained headway distribution;} \]
\[ \delta_1 = \text{the minimum headway in the free headway distribution;} \]
\[ \delta_2 = \text{the minimum headway in the constrained headway distribution;} \]
\[ k = \text{an index that denotes the degree of non-randomness in the constrained headway distribution.} \]

The proposed model, of course, is non-linear. The exponential component of the distribution describes the free (unconstrained) headways in the traffic stream, and the Erlang component describes the constrained headways. It is a very flexible model that can decay to a simple exponential function, to an Erlang function, or to a hyper-exponential function as might be required by the traffic situation. It is likely, however, that a traffic stream will always contain both free and constrained vehicles; and that the general form of the hypererlang function will be required in order to affect an adequate description of the composite headways.

The parameters of the hypererlang function were evaluated for data sets obtained from the 1965 Highway Capacity Manual and from a 1967 Purdue University research project. The evaluation process was affected in two steps. Initial estimates for the parameters of the model were obtained using a rational subdivision technique. By definition the subdivision of constrained headways does not overlap the upper region of the subdistribution of free headways. Because of this, the upper region of the composite function describes only free headways. Thus it was possible to obtain estimates of the parameters of the free-headway subdistribution by fitting the best translated exponential function to the appropriate portion of the headway data. After the initial estimates of the parameters of the free-headway subdistribution had been obtained, it was possible to compute and remove the effect from the composite
data function. The residual that remained formed the sub- 
distribution of constrained headways. The mean of these con- 
strained headways was readily computed, and initial estimates 
of the remaining model parameters were obtained by fitting the 
best Erlang function to the residual subdistribution. The 
appropriate k value for each Erlang function was found from 
a graphical comparison of the residual distribution with 
standardized Erlang functions.

The initial estimates for the parameters were then refined 
using the method of non-linear least squares. In particular, 
the Marquardt Algorithm was used. This algorithm employs the 
method of steepest descent and the method of Gauss to converge 
on a set of parameters that tend to minimize the sum of squares 
of the deviations of the observed headway distribution from the 
theoretical hyperlax distribution.

The proposed hyperlax model proved to be a sound des- 
criptor of the reported headways for volumes ranging from 
about 150 vehicles per hour to about 1050 vehicles per hour. 
However, it should be substantiated and evaluated for a wide 
range of traffic and roadway conditions. During the conduct 
of any future research, careful attention should be given to 
proper flow rate monitoring and to proper data stratification 
to reflect the variations in roadway characteristics that are 
caused by variations in traffic and roadway conditions.

Descriptors: Chinini, Dawson, Probability Distributions, 
Two-Roadway Distributions, Two-lane highway 
models.
THE HYPERLAPIS PROBABILITY DISTRIBUTION--
A GENERALIZED TRAFFIC HEADWAY MODEL

INTRODUCTION

Need for Research

Headways, or time spacings between successive vehicles, are one of the basic characteristics essential to the description of a traffic stream. Although they are seemingly a simple aspect of traffic flow, they can be used collectively as an index of available stream capacity, or as an index of the level of stream congestion. In one of the first extensive studies of traffic headways, reported in the 1950 Highway Capacity Manual, Normann utilized the headway distribution as a basis for describing the level of service available in a traffic stream(1).

It is also apparent that at any point in the traffic lane the vehicular arrival rate is regulated by the sequence of headways between successive vehicles in the stream. In selecting a headway, a driver is simultaneously establishing a flow rate for his vehicle. Normally, the time weighted mean of these individual observations is reported as the average flow rate for the stream.

As a consequence of the importance of headways as a traffic flow descriptor, considerable effort has been expended for the purpose of developing a sound, general purpose headway model. Most of this effort has been directed toward fitting
general mathematical functions to observed headways distributions. Of the many probability distributions that have been proposed, the negative-exponential function, the hyper-exponential function, the Erlang function, and the log-normal function are the better known. No one of these, however, has been found completely adequate as a general purpose headway generator.

Previous Research

The earliest research attempts were directed toward fitting the negative exponential function to observed headway data (5, 6, 12). And although this model proved to be adequate for low volume streams, it is not conceptually sound. It is based upon the assumptions that successive events of concern are independent random occurrences, and that they exist over a \((0, \infty)\) range. Several studies have demonstrated that successive vehicles are not independent of each other. In an early research report Hermann showed that the relative speeds of successive vehicles were dependent upon the headway between the vehicles (9). More recent research of the car-following phenomenon essentially has proved intervehicular dependence (8). Of course the existence of headways over the \((0, \infty)\) range is a physical impossibility. There is a minimum headway in a traffic stream; this minimum is related to the length of the lead vehicle, to the minimum
intervehicular spacing demanded by the trailing vehicle, and to the speed and acceleration of the trailing vehicle.

Some researchers have proposed a negative exponential function with a translated axis to reflect the existence of a real minimum headway (2), but this modified model does not fully compensate for intervehicle dependence.

In an early research study Schuhl suggested the hyper-exponential model (16). Basically it is a linear combination of a negative exponential function and a translated negative exponential function. The translated component was proposed as a descriptor of the headways between constrained vehicles, and the simple exponential component was proposed as a descriptor of the free headways. Kell later modified Schuhl's model so that both the free and the constrained headways were represented by translated functions (11). The proposed modification was rational due to the fact that every vehicle is physically restricted from traveling at a zero (0) headway. With this modification, Kell was able to obtain good model fits for volumes up to about 700 vehicles per hour. In a recent study, however, Sword was able to fit the hyper-exponential function as it was originally proposed by Schuhl, over the same volume range (17).

In an Australian research study Buckley proposed another version of the compound headway model, which he referred to as a semi-random model (1). He supports Schuhl's theory that
a traffic stream is made up of free and constrained vehicles, and like Schuhl proposes that the negative-exponential function be used to describe the free headways. However, Buckley used a normal distribution to describe the constrained headways.

Other researchers have proposed another function. In an attempt to describe the apparent decay of randomness at higher volumes they have suggested the so-called Erlang distribution (7, 15). By varying the model parameters it is possible to simulate time spacings between events that vary from being completely random to being completely non-random or uniform. It is also possible to translate the axis of the Erlang function to take into account the physical constraint that prevents zero and near zero spacings. Nevertheless, there has been little success in fitting this model, except at high and low volume extremes, where the major portion of the headways are either restricted or free, respectively. The Erlang model just is not flexible enough to afford simultaneous descriptions of both free and constrained headways.

More recently Greenberg suggested the log-normal function as a headway model (4). The log-normal function, however, is merely a mathematical transformation that tends to approximate several members of the family of Erlang functions.
Scope of Study

The limited success that has been encountered in attempts to isolate a general purpose headway model can perhaps be attributed to the use of deductive research techniques. Known mathematical models have been proposed, and the parameters of these models have been determined by model fitting techniques.

A more direct study approach is proposed. It involves a methodology that is best described as model evolution, and includes the following steps:

1. Identification of headway properties that are essential for a sound, realistic description of traffic flow past a point;
2. Construction of mathematical micro-components to simulate critical headway properties;
3. Integration of the micro-components into a general mathematical headway generating function;
4. Determination of numerical values for various model parameters; and
5. Evaluation of the model,
Identification of Critical Properties

Two important headway properties have been identified in previous research studies. In the first place it has been established that there are at least two types of vehicles in a traffic stream (11, 16). For descriptive purposes these types are referred to as free vehicles and constrained vehicles. Free vehicles are those that are not under the influence of other vehicles in the traffic stream. For the most part this condition exists when the headway from the free vehicle to preceding vehicles is of "adequate" duration; when the free vehicle is able to pass so that it does not have to modify its time-space trajectory as it approaches preceding vehicles; or when a passing vehicle has sustained a positive relative speed after the passing maneuver so that the free vehicle is still able to operate as an independent unit. Constrained vehicles, of course, are those that are under the influence of other vehicles in the stream. It has also been observed that the balance between these free and constrained vehicles varies with the flow rate of the traffic stream. As the flow rate increases, the proportion of free vehicles decreases and the proportion of constrained vehicles increases (11).

In the second place, it has been established, both by observation and rationalization, that there is a real minimum
headway in a traffic stream that is related to the size and the finite velocity of the vehicle (14). If the traffic stream is a single-lane stream, and headways are measured just between successive vehicles in that stream, there must be real minimums for both the free and constrained headway distributions. If the headways are also measured between the free vehicles occupying the adjacent lane during a passing maneuver, the lower limit for the free headway distribution is zero (0).

Construction of mathematical micro-components

Each of the micro-aspects of a headway distribution can be simulated by an appropriate mathematical function. Because of its random nature, the distribution of free vehicle headways is readily simulated by a negative-exponential distribution of the form,

$$P_f(t) = e^{-β_1 t}$$

where:

- $P_f(t)$ = the probability that a free headway is equal to or greater than $t$;
- $β_1$ = the free vehicle flow rate; and
- $t$ = any time duration.
When the headways are measured on a lane by lane basis, without consideration for the headways of passing vehicles occupying the adjacent lane, minimum headway limits can be simulated by a boundary condition on the generator. Mathematically, the boundary is effected by translating the axis of the function so that the model takes the form,

\[ P_x(t) = \begin{cases} 1, & 0 \leq t \leq \delta_1 \\ \frac{1}{\gamma_1} \left( t - \delta_1 \right)^{\gamma_1 - 1}, & \delta_1 \leq t < \infty \end{cases} \]

where:
- \( P_x(t) \) = the probability that a free headway is equal to or greater than \( t \);
- \( t \) = any time duration;
- \( \delta_1 \) = the minimum free headway; and
- \( \gamma_1 \) = the average free headway.

The headways between constrained vehicles in a traffic stream tend to be non-random, and in some instances appear to approach uniformity. Non-random phenomena, ranging from phenomena that are completely random to phenomena that are completely uniform, are readily simulated by an Erlang
function. Mathematically the Erlang function takes the form,

\[ P_c(t) = e^{-k\beta_2 t} \sum_{x=0}^{k-1} \frac{(k\beta_2 t)^x}{x!} \]

where:

- \( P_c(t) \) = the probability that a constrained headway is equal to or greater than \( t \);
- \( k \) = an index that indicates the degree of non-randomness in the constrained headway distribution;
- \( \beta_2 \) = the constrained vehicle flow rate; and
- \( t \) = any time duration.

Of course the lower time bound on the constrained distribution is a real limit greater than zero (0). To reflect this, the Erlang function was modified to the form,

\[ P_c(t) = 1 \quad : 0 \leq t \leq \delta_2 \]

\[ P_c(t) = e^{-(t-\delta_2)/(\gamma_2-\delta_2)} \sum_{x=0}^{k-1} \frac{(t-\delta_2)^x}{x!} \quad : \delta_2 \leq t \leq \infty \]
where:

\( P_c(t) \) = the probability that a constrained headway is equal to or greater than \( t \);

\( k \) = an index that indicates the degree of non-randomness in the constrained headway distribution;

\( \delta_2 \) = the minimum headway in the constrained headway distribution;

\( \gamma_2 \) = the average headway in the constrained distribution; and

\( t \) = any time duration.

In order to reflect the relative proportion of the total vehicles in the stream that are either free or constrained, it is only necessary to introduce linear coefficients, \( a_1 \) and \( a_2 \), before each of the component functions. The \( a_1 \) denotes the proportion of free vehicles in the traffic stream, and \( a_2 \) (\( a_2 = 1 - a_1 \)) denotes the proportion of constrained vehicles in the stream.

The Complete Model

The general purpose headway model is obtained by forming a linear combination of the free and constrained components.
The integrated model takes the form,

\[ P(h > t) = a_1 e^{-\frac{(t - \delta_1)}{(\gamma_1 - \delta_1)}} + a_2 \sum_{k=1}^{x} e^{-\frac{1}{(\gamma_2 - \delta_2)}} \frac{(t - \delta_2)}{(\gamma_2 - \delta_2)^x} \]  

for \( x \geq 0 \).

This model has been proposed by Buckley (1) and by Dawson (3), but it has not been researched. Dawson suggested that it be called the hyper-Erlang function. For the purpose of this paper, however, the name has been shortened to the "hyperlang" function.

The hyperlang function is a very general model; in fact it includes the negative exponential, the hyper-exponential, and the Erlang functions as special cases. This can be illustrated with schematic comparisons of the several functions.

An exponential headway simulator is depicted in Figure 1(a). From a continuous queue at the entrance to a holding area, individual vehicles are randomly released. If
(a) Exponential Headway Simulator

\[ t = 1/\theta \]

(b) Erlang Headway Simulators

\[ k \text{ exponential holding areas in series} \]

(c) Hyper-exponential Headway Simulator

\[ k \geq 2 \text{ exponential holding areas in parallel} \]

FIGURE 1 -- SCHEMATIC DIAGRAM OF EXPONENTIAL, ERLANG AND HYPER-EXponential HEADWAY SIMULATORS
the vehicles are metered out of the holding area at a mean flow rate of \( \beta \) vehicles per unit time, the resulting headways will follow an exponential distribution with a mean of 

\[ \bar{\tau} = 1/\beta. \]

Figure 1(b) depicts an Erlang headway simulator. From a continuous queue at the entrance to a holding area made up of several phases in series, one vehicle at a time is allowed to enter the area. The entering vehicle goes through phase (1) where the holding times are exponentially distributed. Upon leaving phase (1), the vehicle enters phase (2), phase (3), \ldots, phase (k-1), and phase (k), each with a mean rate of release of \( k \beta \). The distribution of times between departures of the vehicles from phase (k), and therefore from the overall holding area, are described by an Erlang distribution with a mean time, \( \bar{\tau} = 1/\beta. \) Although the holding area is compounded, it should be considered as a single area because only one vehicle at a time is allowed in it. That is, the \((n+1)\) vehicle cannot enter phase (1) until the \( n \) vehicle has departed from phase (k).

Hyper-random or hyper-exponential headways can be generated by the simulator depicted in Figure 1(c). From a continuous queue at the entrance to a holding area made up of \( k \) channels in parallel, one vehicle at a time is allowed to enter the area. As a vehicle enters, it is assigned to one of the \( k \) channels at random, but on the average it is
FIGURE 2 — HYPERLAMC HEADWAY SIMULATOR
area channel leaving a hyper-exponential system. It is likely, however, that a traffic stream will always contain both free and constrained vehicles; and that the general hyperbolic function will be required in order to effect a complete description of the composite headways.
NUMERICAL EVALUATION OF MODEL PARAMETERS

Selection of Data

The parameters for the hyperlong headway model were evaluated for one-lane flows on two-lane, two-way roadways. Two separate analyses were conducted. One was based on the headway data for two-lane roadways reported in the 1965 Highway Capacity Manual (10), and the other analysis was based on data from a recent Purdue University study (17). Plots of the cumulative headway distributions depicting the data sets obtained in these two studies are presented in Figures 3 and 4, respectively. The several plots obtained from the 1965 Highway Capacity Manual (Figure 3) were purported to represent flow rates ranging from 150 to 1050 vehicles per hour in increments of 100 vehicles per hour, and in general they tend to form a uniform family. There are some apparent irregularities in the higher volume curves, at the longer headways; but these irregularities are not as serious as they appear to be. In this region of the plots, minor irregularities are greatly exaggerated by the logarithmic probability scale.

The data obtained from the Purdue study (Figure 4) is purported to have been collected at flow rates ranging from approximately 150 to 950 vehicles per hour, in increments of approximately 100 vehicles per hour; but as can be seen from the headway plots, the flow rates apparently were not
monitored accurately. The 251 vehicles per hour curve tends to meander from the 353 vehicles per hour curve at low headways up to the 151 vehicles per hour curve at longer headways. The 450, 547, and 651 vehicles per hour curves are all rather closely spaced; in fact, the 450 and 547 vehicles per hour curves actually cross each other. The three higher volume plots are also somewhat peculiar. The 746 and 836 vehicle per hour curves tend to be quite similar; whereas the 957 vehicle per hour curve tends to diverge from the family. This lack of uniformity between the curves indicates that the reported flow rates are averages of extreme rates within the implied flow ranges. The composite headway distribution that is formed by combining the headways from two streams with different flow rates, however, is quite different from the distribution of headways from a stream that flows uniformly at the same apparent rate. These criticisms do not invalidate the data sets; they merely emphasize the importance of properly monitoring the traffic stream while making headway measurements.

These data sets were particularly appropriate for the hyperlavg evaluation studies. The hyperlavg function was rationally constructed to effect a sound descriptor for traffic headways. Each of the seven function parameters has physical significance. The consistent and uniform nature of the 1965 Highway Capacity Manual data afforded the opportunity both to
numerically evaluate the parameters, and to observe the relationships between the parameters and the corresponding flow rates. On the other hand the erratic nature of the Purdue University data afforded the opportunity to observe the power of the hyperlang function as a descriptive headway model. As suggested earlier these non-uniform data sets are very likely arbitrary combinations of free and constrained headways that were measured during periods of unsteady flow. The facility (or lack thereof) to describe these arbitrary distributions is an indication of the power of the hyperlang function. It was not likely, however, that the model parameters would form a consistent relationship with the reported flow rates.

Determination of Parameters

Parameter evaluation was affected in two steps. In the first step, initial estimates for the parameters of the hyper-lang function were obtained using a rational subdivision technique. These initial estimates were then refined in a second step using the method of non-linear least squares. By definition the subdistribution of the constrained headways cannot overlap the upper region of the subdistribution of the free headways. Because of this, the upper regions of the cumulative plots of Figures 3 and 4 describe only free headways. Thus, it was possible to obtain initial
estimates for the parameters of the free headway subdistribution by fitting the best translated exponential function to the appropriate portion of the headway data. The cumulative data plots of Figures 3 and 4 proved to be of value in establishing the free headway cut-off points. These graphs are constructed with logarithmic probability scales so that the free portions of the distributions tend to plot as straight lines.

After the initial estimates for the parameters of the free headway subdistribution had been obtained, it was possible to compute and subtract this subdistribution from the overall headway distribution. The residuals that remained formed the subdistribution of constrained headways. The mean of these constrained headways was readily estimated, and initial estimates for the remaining model parameters were obtained by fitting the best Erlang function to the subdistribution. The appropriate $k$ values for the Erlang distributions were found from graphical comparisons of the residual distributions with standardized Erlang functions. Two such comparisons are illustrated in Figure 5. The residual distribution for the 250 vph data set from 1965 Highway Capacity Manual apparently follows the $k=2$ curve; and the residual distribution for the 957 vph data set from the Purdue study was best approximated by the $k=6$ curve.
The technique for obtaining the initial estimates for the parameters of the hyperlang function involved trial and measurement. Several combinations of minimum headway limits for the free and constrained distributions were tried. For each of these combinations the remaining model parameters for the free subdistribution were estimated by the method of least squares; and the remaining parameters for the constrained subdistribution were estimated by computational and by graphical methods. The quality of each of the resulting hyperlang models was reported as the percentage of the total variation within the data that was removed by the model. This measurement, of course, is equivalent to \( R^2 \), the square of the multiple correlation coefficient. In each case the set of models parameters that yielded the highest \( R^2 \) value was selected for further refinement.

In the second step of the parameter evaluation process the initial parameter estimates were further refined using Marquardt's algorithm for non-linear least-squares analysis (13). This algorithm employs the method of steepest descent and the method of Gauss to converge on a set of parameter values that tend to minimize the sum of squares of the deviations of the observed headways about the theoretical hyperlang headway function.
Highway Capacity Manual Study

The hyperlang models that were selected to describe the headway distributions reported in the 1965 Highway Capacity Manual are plotted as a family in Figure 6. The corresponding data sets are not superimposed on the individual curves due to their proximity; but the individual functions were graphed separately, along with the corresponding sets of raw data. The plot for the 250 vph flow rate is presented as an example in Figure 7. Graphs for all of the flow rates are presented in Appendix A. The model parameters that describe these hyperlang plots are tabulated in Table 1, and it is interesting to note that all of the $R^2$ values fall between .9991 and .9998.

Several other interesting observations can be made from this tabulation. The $k$ indices for the Erlang components took on a value of two (2) at all levels of flow. Prior to the study, however, it had been hypothesized that the $k$ values would increase as the flow rate increased (or in other words, the constrained headways would tend to become more uniformly distributed at higher volumes). This hypothesis was based on an assumption that the increase in the number of constrained vehicles in the traffic stream would be compounded by simultaneous increases in the flow rate and in the proportion of constrained vehicles in the stream.
Data From
1965 Highway
Capacity Manual

\[ E^2 \approx 0.9999 \]

**Figure 7: Hyperbolic Headway Model**

Volume: Z.C. V. F. H.
<table>
<thead>
<tr>
<th>Monitored Flow Rate</th>
<th>Computed Flow Rate</th>
<th>$r^2$</th>
<th>$a_1$</th>
<th>$y_1$</th>
<th>$\delta_1$</th>
<th>$k$</th>
<th>$a_2$</th>
<th>$y_2$</th>
<th>$\delta_2$</th>
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<tbody>
<tr>
<td>150</td>
<td>166</td>
<td>0.9991</td>
<td>0.54</td>
<td>32.63</td>
<td>0.75</td>
<td>2</td>
<td>0.36</td>
<td>2.17</td>
<td>0.75</td>
</tr>
<tr>
<td>250</td>
<td>249</td>
<td>0.9995</td>
<td>0.55</td>
<td>24.62</td>
<td>0.75</td>
<td>2</td>
<td>0.45</td>
<td>2.12</td>
<td>0.75</td>
</tr>
<tr>
<td>350</td>
<td>344</td>
<td>0.9997</td>
<td>0.48</td>
<td>19.48</td>
<td>0.75</td>
<td>2</td>
<td>0.52</td>
<td>2.12</td>
<td>0.75</td>
</tr>
<tr>
<td>450</td>
<td>445</td>
<td>0.9995</td>
<td>0.43</td>
<td>15.06</td>
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<td>0.57</td>
<td>2.14</td>
<td>0.70</td>
</tr>
<tr>
<td>550</td>
<td>528</td>
<td>0.9992</td>
<td>0.38</td>
<td>14.23</td>
<td>0.75</td>
<td>2</td>
<td>0.62</td>
<td>2.19</td>
<td>0.66</td>
</tr>
<tr>
<td>650</td>
<td>625</td>
<td>0.9995</td>
<td>0.36</td>
<td>12.02</td>
<td>0.75</td>
<td>2</td>
<td>0.64</td>
<td>2.19</td>
<td>0.61</td>
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<tr>
<td>750</td>
<td>726</td>
<td>0.9997</td>
<td>0.33</td>
<td>10.49</td>
<td>0.75</td>
<td>2</td>
<td>0.67</td>
<td>2.22</td>
<td>0.56</td>
</tr>
<tr>
<td>850</td>
<td>813</td>
<td>0.9998</td>
<td>0.30</td>
<td>9.60</td>
<td>0.75</td>
<td>2</td>
<td>0.70</td>
<td>2.22</td>
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</tr>
<tr>
<td>950</td>
<td>926</td>
<td>0.9997</td>
<td>0.26</td>
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<tr>
<td>1050</td>
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Although the k values did not vary as had been anticipated, the tabulated $a_1$ and $a_2$ values indicate that the proportion of constrained vehicles in the stream do increase in a direct relation with the flow rate. The proportion of constrained vehicles varied from 36 percent at a flow rate of 150 vph to 79 percent at a flow rate of 1050 vph. No satisfactory explanation has been found for the apparent lack of conformance with a priori hypotheses. It is possible, however, that distinctive properties in the several headway distributions were averaged out in combining data without regard for stratification to reflect variations in traffic and environmental conditions.

Another interesting characteristic was observed between the minimum headways for the free vehicle distributions. The lower bounds for these distributions ($\delta_1$) clustered in a random pattern about a central value of approximately 0.75 seconds. For this reason $\delta_1$, the parameter representing the minimum headway between free vehicles, was held constant at a value of 0.75 seconds for all flow rates. This appears to be rational. At a flow rate of 150 vph, a time headway of 0.75 seconds is indicative of an inter-vehicular spacing of approximately 35 feet; at a flow rate of 1050 vph, it is indicative of an inter-vehicular spacing of approximately 15 feet. Of course, these are the lower bounds for the spacings between all free vehicles.
as a family in Figure 8. The corresponding data sets were
not superimposed on the individual curves because of their
proximity; but the individual functions were graphed separately,
along with the corresponding sets of raw data. The plot for
the 957 vph flow rate is presented as an example in Figure 9.
Graphs for all flow rates are presented in Appendix B.

The several headway functions that are presented in
Figure 8 do not seem to constitute a very uniform family; but
as can be seen in Figure 9 and in the plots of Appendix B,
the hyperlang functions do fit the reported data very well.
The models parameters that describe the hyperlang plots of
Figure 8, and the \( R^2 \) values that are associated with each of
the relationships, are tabulated in Table 2. Again, it is
interesting to note that the \( R^2 \) values are very high. The
two lowest values are .9979 and .9988, but the rest of the
values range between .9995 and .9998.

In general the results that were obtained using the
Purdue data more nearly correspond to a priori hypotheses.
The \( k \) indices for the Erlang components vary directly with
the flow rates. They range from \( (k=1) \) at a flow rate of
158 vph, to \( (k=6) \) at a flow rate of 957 vph. There also
seems to be a significant relationship between these \( k \) values
and the curve cluster patterns in Figure 8 and more parti-
cularly in Figure 4. The several headway distributions plotted
Figure 8 - HyperLang Headway Distributions for Purdue Research Project Data
Data From
U.S. 52 - Bypass
Lafayette, Indiana

$R^2 = .9027$

Figure 9 -- HYPERLANC HIGHWAY MODEL
VOLUME: 857 F. H.
TABLE 2

HYPERLANE MODEL PARAMETERS (Furui University Data)

<table>
<thead>
<tr>
<th>Computed Flow Rate</th>
<th>Monitored Flow Rate</th>
<th>$V_2$</th>
<th>$V_1$</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.58</td>
<td>155</td>
<td>1.02</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>2.05</td>
<td>146</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>2.51</td>
<td>130</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>3.03</td>
<td>118</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>3.53</td>
<td>107</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>4.07</td>
<td>98</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>4.60</td>
<td>88</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>5.27</td>
<td>79</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>6.10</td>
<td>70</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>7.04</td>
<td>61</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>8.11</td>
<td>53</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>9.35</td>
<td>46</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>10.74</td>
<td>39</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>12.33</td>
<td>33</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>14.12</td>
<td>27</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>16.18</td>
<td>21</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>18.56</td>
<td>16</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>21.26</td>
<td>12</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>24.23</td>
<td>9</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>27.49</td>
<td>6</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>31.00</td>
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<td>1.01</td>
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<td>1.87</td>
</tr>
<tr>
<td>34.75</td>
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<td>1.01</td>
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<td>1.87</td>
</tr>
<tr>
<td>38.74</td>
<td>2</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
<tr>
<td>42.97</td>
<td>1</td>
<td>1.01</td>
<td>0.51</td>
<td>1.87</td>
</tr>
</tbody>
</table>

$\begin{array}{c|c|c|c|c}
\text{Computed Flow Rate} & \text{Monitored Flow Rate} & V_2 & V_1 & k \\
1.58 & 155 & 1.02 & 0.51 & 1.87 \\
2.05 & 146 & 1.01 & 0.51 & 1.87 \\
2.51 & 130 & 1.01 & 0.51 & 1.87 \\
3.03 & 118 & 1.01 & 0.51 & 1.87 \\
3.53 & 107 & 1.01 & 0.51 & 1.87 \\
4.07 & 98 & 1.01 & 0.51 & 1.87 \\
4.60 & 88 & 1.01 & 0.51 & 1.87 \\
5.27 & 79 & 1.01 & 0.51 & 1.87 \\
6.10 & 70 & 1.01 & 0.51 & 1.87 \\
7.04 & 61 & 1.01 & 0.51 & 1.87 \\
8.11 & 53 & 1.01 & 0.51 & 1.87 \\
9.35 & 46 & 1.01 & 0.51 & 1.87 \\
10.74 & 39 & 1.01 & 0.51 & 1.87 \\
12.33 & 33 & 1.01 & 0.51 & 1.87 \\
14.12 & 27 & 1.01 & 0.51 & 1.87 \\
16.18 & 21 & 1.01 & 0.51 & 1.87 \\
18.56 & 16 & 1.01 & 0.51 & 1.87 \\
21.26 & 12 & 1.01 & 0.51 & 1.87 \\
24.23 & 9 & 1.01 & 0.51 & 1.87 \\
27.49 & 6 & 1.01 & 0.51 & 1.87 \\
31.00 & 4 & 1.01 & 0.51 & 1.87 \\
34.75 & 3 & 1.01 & 0.51 & 1.87 \\
38.74 & 2 & 1.01 & 0.51 & 1.87 \\
42.97 & 1 & 1.01 & 0.51 & 1.87 \\
\end{array}$
The limiting headways that were derived for the constrained vehicles ($\delta_2$) also tend to decrease as the flow rate increases. This latter inverse relationship is in agreement with the results that were obtained in the analysis of the Highway Capacity Manual Data.
CONCLUSIONS AND RECOMMENDATIONS

The hyperlang function is apparently a sound model for describing the headways in single-lane flows on two-lane, two-way roadways. It is a very flexible model that can decay to a simple exponential function, to an Erlang function, or to a hyper-exponential function. It is likely, however, that a traffic stream will always contain both free and constrained vehicles; and that the general form of the hyperlang function will be required to effect an adequate description of the composite headways.

The parameters of the hyperlang model were evaluated separately for data sets obtained from the 1965 Highway Capacity Manual and from a recent Purdue University research project. In both instances the proposed model proved to be an excellent descriptor of the reported headway distributions. There is need, however, for a more comprehensive research study of the hyperlang headway model. Some of the parameter relationships that were observed in this preliminary study were not consistent for both data sets; and it appears that the differences are the result of inconsistencies in the original research data. In future studies careful attention should be given to proper flow rate monitoring while making headway measurements; and careful attention should also be given to proper data stratification, during the analysis, to reflect variations in traffic and roadway conditions.
REFERENCES


APPENDICES
APPENDIX A

Plots of Hyperlang Headway Models And Cumulative Headway Data

(1965 Highway Capacity Manual Data)
FIGURE A3  HYDRAULIC HEADWAY MODEL
VOLUEM: 350 V. P. M.

Data From
1965 Highway Capacity Manual

$R^2 = 0.9997$
Date: June 1966

1965 Highway Capacity Manual

$R^2 = .9966$

Figure A10: Interlacing Heuristic Model

Volume: 1850 vph
APPENDIX B

Plots of Hyperlang Headway Models and Cumulative Headway Data

(Purdue University Research Project Data)
FIGURE B4 -- HYPERLAM BACKGROUND MODEL
VOLUME: 450 V. P. H.
Data From
U.S. 32 - Bypass
Lafayette, Indiana

FIGURE D1 -- INTERLAM BENDY TESTING
VOLUMES: 247 G. F. M.