REDUCTION OF AI-PARENT AGGREGATE VARIATION THROUGH IMPROVED SAMPLING

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Research carried out through Joint Highway Research Council of the University of Connecticut and the Connecticut Department of Highways with the cooperation of the Connecticut Crushed Stone Association and the National Crushed Stone Association.
Synopsis

This study establishes an improved method of sampling aggregates in bulk. The method is simple, economically feasible, and more consistent than those used in the past. The method will, for the first time, give the materials engineer a reliable measure of the material used and permit accurate design of mixes. The improved reliability of the samples will reduce the frequency of erroneous work stoppages and thus benefit the supplier.

The method is statistically oriented, based on the statistical principle that extreme values are less probable in large samples. In principle, a truck load of material is the sample. Such a sample, whether drawn from bins or loaded from stock piles, will more nearly represent the bulk of the material. The frequency of sampling can be reduced, as statistically the frequency is interrelated with standard deviation. A reduction in deviation will permit reduced sampling.

Obviously, entire truck loads can not be screened as samples. Therefore, it was necessary to determine the number of samples from a single truck required to give a high statistical confidence that the mean of the samples truly represents the truck load. Finally, in order to keep the volume of testing within bounds, the samples from one truck are combined into one sample which reduces the testing by mechanically averaging the individual samples.
During the spring of 1965, the University of Connecticut - Connecticut Highway Department Joint Highway Research Committee considered the question of aggregate characteristic as related to sampling point. The preliminary draft of the report for a recent national study conducted under the auspices of the National Cooperative Highway Research Program implied strong variations but no distinct correlation with stage of construction.

Plans to follow the gradation of aggregate through construction stages was shelved until the reasons for the poor success referred to above could be formulated. The most probable cause appeared to be the method of sampling. Sampling has always been the most questionable part of quality control. If the sample does not accurately represent the material, the test data is useless. Any trend existing in aggregate gradation throughout construction stages could readily be masked by the variability introduced through poor sampling methods.

Additional benefits would accrue to the highway department if improved sampling methods could be developed. To use statistical control, the standard deviation and mean of the material must be determined from intensive sampling. It is important that the samples be representative of the material. Otherwise, the standard deviation found will be that of the samples and not that of the material. It is equally important that for routine control the samples compared to the standard deviation must be representative. As the sampling frequency is reduced through the use of statistics, the importance of each sample is magnified.

This problem is further complicated by the question of location of the sampling station. Should the material be sampled at the time
of production, storage, handling or final position in the work? After selecting a sampling location, the method of sampling must be determined. In the past, most sampling has been performed at the surface of large quantities of material. The most frequent location being the surface of stock piles. It has been widely recognized that the surface of a stock pile may differ from the bulk of the material.

This paper presents work carried out to develop an accurate means of collecting representative samples of aggregates. Not only should the samples represent the material, but the procedure should be simple, applicable to most situations, rapid, and economically feasible.

Procedure

At the conception of the study, it was envisioned that the ideal sampling points would be at the discharge from the screen in the screen house of the aggregate plant. Consequently, a large modern screening plant was selected and intensive sampling carried out at the screen discharges over a four week period. Eighty to one hundred pound samples were taken at hourly intervals from the five sizes of commercial aggregate being prepared in this plant. The samples were divided into two portions and processed through two different laboratories using Gilson sieves.

The results of this work were examined statistically and comparisons made to routine samples taken from the same production for the State Laboratory. The sieve analysis at the screens revealed variations due to loading or the screens and moisture in the incoming crushed stone. Such variations on individual screens of parallel decks are of interest to the plant but not serious to the user. See Appendix 2.
The standard deviations of the percent passing various sieves are listed in Table 1 for the two series of samples. The significant differences shown between the two groups of samples indicated that the routine sampling did not adequately represent the material and that an improved method should be developed.

Table 1  STANDARD DEVIATION OF SCREEN HOUSE STUDY SAMPLES AND ROUTINE FIELD SAMPLES.

<table>
<thead>
<tr>
<th>3/8&quot; Stone</th>
<th>Sieve 3/4&quot;</th>
<th>1/2&quot;</th>
<th>3/8&quot;</th>
<th>#4</th>
<th>#6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Routine</td>
<td>0.00</td>
<td>1.11</td>
<td>8.06</td>
<td>2.99</td>
<td>1.11</td>
</tr>
<tr>
<td>Study at Screens</td>
<td>0.00</td>
<td>0.63</td>
<td>5.58</td>
<td>0.47</td>
<td>0.22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3/4&quot; Stone</th>
<th>Sieve 1&quot;</th>
<th>3/4&quot;</th>
<th>1/4&quot;</th>
<th>3/8&quot;</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Routine</td>
<td>0.00</td>
<td>1.22</td>
<td>11.11</td>
<td>2.92</td>
<td>1.45</td>
</tr>
<tr>
<td>Study at Screens</td>
<td>0.00</td>
<td>0.63</td>
<td>4.50</td>
<td>0.38</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Note: Values are standard deviations of percents passing sieves.

In most plants the amount of labor required for sampling at the screen decks would be high. Parallel screens require multiple samples, for which the proper weighting when averaging is not obtainable.

Physically, most plants are not planned for such an operation and differently shaped sampling containers would be needed for each deck in each plant. All samples would have to be carried down catwalks from the upper portions of the plants. In addition, the variation of the material from any one screen is greater than that of the combined plant production of that size.

All or the plants observed delivered the material from the screen decks into large storage hoppers or aggregate bins. These bins tend to act as tempering chambers damping out short term variations in the aggregate gradation. The way in which aggregate flows through an active hopper is exceedingly complex. As the material is not used in the condition it comes from the screen deck, it is apparent that samples drawn
from the storage bins would more nearly represent the gradation of the material delivered to the user. The material in a storage bin is not removed in 6" layers and laid down as such on the roadway but is re-mixed as the material flows from the hopper into the next container. It is possible to construct a sampler to be located immediately below the discharge gates of the storage bin. However, it is quickly realized that a cubic foot of aggregate coming from the discharge gate will be distributed over an appreciable area within the truck or railcar receiving the material from the hopper and when dumped from this container will be rearranged once more. Consequently, it would seem desirable to sample the material delivered from a truck as being more nearly representative of the material delivered to the user.

An observer, watching the flow of aggregate through a bin or hopper can readily discern the complexity of the motion but he would be hard pressed to describe the action. The proposal of intermittently spray painting the stream of aggregate entering a bin was made. In order to locate the colors when dispersed, a ton or so of aggregate should be painted. This material might easily contaminate several hundreds of tons of aggregate before all treated stone cleared the bin. The result would be only a visual impression of the flow action under the loading and discharge rates at the time of treatment and observation. The probability of a successful trial did not appear good. Consequently, this proposal was tried on a small scale model. Figure la shows a hopper of horizontally stratified material, which was then dropped into a simulated truck figure lb which in turn was dumped onto the ground figure lc. The colored aggregate was spray painted ball bearings. Figure 2 represents a vertically stratified material treated in a similar manner. Of course, the particle size, shape and bin dimensions
FIG. 1 HORIZONTAL SEGREGATION

FIG. 2 VERTICAL SEGREGATION
are out of proportion so no quantitative inference should be made but it does appear obvious that local variations within the bin are re-
distributed in subsequent operations. It is equally apparent that a
bin condition of perfectly uniform distribution would not necessarily
result in perfectly uniform distribution in the truck.

Obtaining an aggregate sample from a truck that represents the
gradation of the material is a problem that has long plagued the high-
way industry. For statistical control, each sample size unit of
material must have an equal chance of being selected by the sampler.
A man on a truck with a shovel is not able to sample more than the top
twelve to eighteen inches of aggregate. Large trucks now in use have
a depth of material frequently in excess of six feet. This would mean
that 75% or more of the material in the truck has no possible chance
of being sampled. The procedure frequently proposed is to lay off the
surface of a truck in a grid pattern and then select grid points
randomly. Samples taken in this manner are not truly representative
but are biased in the sense that all come from near the surface of the
material. One might say that they are randomly selected surface samples.

Recent work of the cooperative highway research program has rein-
forced the view that samples to be representative should be taken from
a flowing stream of aggregate. Given a truck loaded with aggregate
and the problem of how to select samples that come from a flowing
stream and from any portion of the truck, it is readily apparent that
raising the body and allowing the stone to run slowly out of the truck
permits meeting these conditions.

In order to find the variation that exists within the material in
one truck load, it was decided to intensely sample truck loads of
aggregate by catching frequent samples of the stone as it came through the tailgate. To expedite collecting samples, trucks with tailgate openings for delivering sand or aggregate to spreaders were utilised. The truck was loaded under the aggregate bins with from 4 to 13 tons of aggregate, then driven a short distance to storage piles and the body raised and the stone allowed to run out through the small gate. It was found possible to take and handle samples at a rate of one a minute. The stone was unloaded at the rate of a ton per minute permitting one sample to be secured for each ton of material. Each sample was bagged and processed as an individual sample. Different size aggregates from 3/8 to 1-1/4 inch were used. Several trucks of each size were sampled and all samples were sieved. The results were processed statistically to find the deviation and mean for each material on each sieve size. Of the 5 sieve sizes used for grading each sample, only the middle size had an significance. Consequently, the statistical work was concentrated on the results of the middle sieve.

It should be recognized that the aggregate processed is being produced to meet the 5 sieve series gradation and, consequently, the plant control methods have been planned to be safe on the second and fourth sieve.

Comparison of standard deviations found for the different sizes of sampling trucks indicate little correlation. Only weak correlation exists with the point within a truck. A summary of the truck loads represented in figures 15, 16, and 17 is given in table 2. Each entry in the table is the standard deviation of the percents passing a given sieve for all samples from one truck.
Table 2  STANDARD DEVIATIONS OF PERCENT PASSING DESIGNATED SIEVES,  
3/4" AGGREGATE

<table>
<thead>
<tr>
<th>Load</th>
<th>1&quot;</th>
<th>3/4&quot;</th>
<th>1/2&quot;</th>
<th>3/8&quot;</th>
<th>#4</th>
<th>Truck No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7,500#</td>
<td>0.00</td>
<td>2.04</td>
<td>4.24</td>
<td>0.15</td>
<td>0.18</td>
<td>1</td>
</tr>
<tr>
<td>14,000#</td>
<td>0.00</td>
<td>1.73</td>
<td>2.03</td>
<td>1.24</td>
<td>0.59</td>
<td>3</td>
</tr>
<tr>
<td>24,000#</td>
<td>0.00</td>
<td>3.28</td>
<td>6.60</td>
<td>1.62</td>
<td>0.59</td>
<td>5</td>
</tr>
</tbody>
</table>

The zero deviation on a one inch sieve is to be expected as the 3/4" stone is processed from material passing the 1" screen in the plant. As 100% passed for all samples, there is no computable deviation. For the other sizes, there appears to be no definite trend in deviation.

Computations were made to determine the number of samples needed to represent a truck. Using standard deviation of the range found for the majority of the material, this being something less than 6%, and assuming a desired confidence level of 95%, three samples per truck would be required. See Appendix 2. Reviewing curves 9, 10, and 11, it appeared that the three samples should come, one from the first third, one from the middle, and one from the last third, if the average of the three is to approach the average of the truck. To verify this, a computer program was prepared which computed the average gradation for three samples selected one from the first third, one from the second third, and one from the last third of the truck.

While the mean of the samples approximated the mean of the combinations, the standard deviation of the combination is substantially lower than that of the original samples. Statistically this would be expected, as the average of three should be better than any one. Throughout the data presented the standard deviation of the combination is in the range of one-third to one-half of the standard deviation of the original samples. Recognizing that the material in a truck is redistributed when
unloaded, the mean and standard deviation of the combination samples should more nearly represent the material than any one sample from the truck.

For each material, one fraction becomes the critical control factor. Considering the gradation results, at the top sieve size 100% of nearly all the aggregate passed. Of course, for this sieve the mean was then 100% and the standard deviation was nothing for all truck loads of material. For the second sieve size of the test series, some differences in mean and standard deviation do appear. In general terms, if the numerical values found are typical, this sieve is not a problem. The third sieve size, which is the first size smaller than the size designation of the aggregate, displayed substantial variation in both mean and standard deviation. Computations were completed on the third sieve sizes only. It can be noted that the deviations for individual truck loads are all of the same general magnitude. It then appears that the deviation is independent of the mean and of the exact value or range of the gradation being produced.

Comparisons of all the tests made on one size of material can be made after certain adjustments. Under Connecticut specifications, stone is acceptable if the percent retained on a given sieve is within a specified range. Consequently, the producers were not attempting to provide stone with a single mean on each sieve. It is illogical to lump all test results to find a single mean and the resulting standard deviation for each sieve.

Noticing that the standard deviation for all producers regardless of mean was about the same, it appears more logical to assume that each producer, if desired, could shift the mean without changing significantly
the standard deviation. This approach is shown in Figures 3, 4, 5, and 6.

In these figures, the mean for each material was computed using all sources. Then the data for each source was shifted to this new mean by transposing.

For Figure 3, the mean of the percent passing the 3" sieve for 1-1/4" stone was computed using all sources. Then the data for each producer was transposed to this mean. That is, the test results for each producer were uniformly increased or decreased by the amount this mean differed from the total mean. Such a numerical shift has the same effect as altering the screening plant to change the mean without affecting the standard deviation. Shifting to a common mean makes overall comparisons possible.

The total bar length (clear plus solid) represents the standard deviation of individual samples within an individual truck. Each group represents one source. The horizontal solid line through each group is the standard deviation of all the individual samples taken from that source. At the extreme right the single bar represents the standard deviation of individual samples from all sources after shifting each source to the common mean.

On the same figure 3, the information computed for three part combination samples is shown. The clear portion of the bars shows the standard deviation of combination samples for each truck. The broken horizontal lines are the standard deviation of each source by combination samples. The single clear bar at the right is the standard deviation of the total combination samples.

For each measure, the use of three part combination samples re-
Fig. 3  STANDARD DEVIATION PER TRUCK LOAD IN PER CENT
Fig. 4 STANDARD DEVIATION PER TRUCK LOAD IN PER CENT
Fig. 5 STANDARD DEVIATION PER TRUCK LOAD IN PER CENT
FIG. 6  STANDARD DEVIATION PER TRUCK LOAD IN PER CENT

3/8" Aggregate  #4 Critical Size
duces the apparent standard deviation substantially. For individual trucks the combination deviation is 1/3 to 1/2 of the individual sample deviation. The total material deviation is reduced by one half.

Figures 4, 5, and 6 are similar charts for other materials and show similar reductions.

As a final verification of sampling improvement, the deviation of the study samples are compared to the test reports of the Highway Materials Laboratory. For each period that sampling was carried out at a certain plant, the test records of the Highway Laboratory were collected. The tabulation of Table #3 summarizes this information and

<table>
<thead>
<tr>
<th>Table #3 STANDARD DEVIATION OF SAMPLES BY SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>First entry</td>
</tr>
<tr>
<td>Second entry</td>
</tr>
<tr>
<td>Third entry</td>
</tr>
<tr>
<td>H/N Laboratory Reports</td>
</tr>
<tr>
<td>Single Samples From Study</td>
</tr>
<tr>
<td>Three Parts Samples from Study</td>
</tr>
</tbody>
</table>

Values are $\sigma$ in % for percent passing critical sieve size.

<table>
<thead>
<tr>
<th>Source</th>
<th>1-1/8&quot; Stone</th>
<th>3/8&quot; Stone</th>
<th>1/2&quot; Stone</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>14.4%</td>
<td>13.04%</td>
<td>5.76%</td>
</tr>
<tr>
<td></td>
<td>9.1%</td>
<td>5.24%</td>
<td>7.09%</td>
</tr>
<tr>
<td></td>
<td>8.19%</td>
<td>3.33%</td>
<td>4.65%</td>
</tr>
<tr>
<td>3.</td>
<td>5.64%</td>
<td>8.36%</td>
<td>3.83%</td>
</tr>
<tr>
<td></td>
<td>4.85%</td>
<td>4.48%</td>
<td>4.08%</td>
</tr>
<tr>
<td></td>
<td>3.37%</td>
<td>2.10%</td>
<td>2.36%</td>
</tr>
<tr>
<td>4.</td>
<td>12.02%</td>
<td>8.55%</td>
<td>4.25%</td>
</tr>
<tr>
<td></td>
<td>5.85%</td>
<td>8.17%</td>
<td>7.12%</td>
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<tr>
<td></td>
<td>2.75%</td>
<td>3.88%</td>
<td>6.86%</td>
</tr>
<tr>
<td>5.</td>
<td>3.46%</td>
<td>10.02%</td>
<td>4.29%</td>
</tr>
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<td></td>
<td>6.87%</td>
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</tr>
<tr>
<td></td>
<td>2.96%</td>
<td>4.29%</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>4.24%</td>
<td></td>
<td>4.15%</td>
</tr>
<tr>
<td></td>
<td>5.69%</td>
<td></td>
<td>5.99%</td>
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<td></td>
<td>4.50%</td>
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<td>Total</td>
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<td>9.90%</td>
<td>4.55%</td>
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<tr>
<td></td>
<td>6.48%</td>
<td>6.74%</td>
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</tr>
<tr>
<td></td>
<td>4.35%</td>
<td>3.45%</td>
<td>4.26%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4.05%</td>
</tr>
</tbody>
</table>
makes easy comparisons possible. Of the 24 plant-materials included in the study, laboratory tests in quantity were available for only 13.

The deviation of the individual samples taken for this study exceeded that for the highway laboratory reports in six of the comparisons and showed an improvement in seven instances. However, the magnitude of the improvements were such as to show an overall reduction from 7.52% to 6.43%.

A sampling method which makes provision for access to a larger volume of material for sampling would be expected to improve the sampling performance. It should be noted that the high deviations were all reduced and the low increased. A conscientious sampler with a shovel at a stock pile must balance sampling of the finer top and coarser fringe. Results can show large variations which could be reduced by changes in sampling. Samples taken in a consistent manner could show a misleadingly low deviation which would be increased by a more random form of sampling.

Comparisons drawn with the three part combination samples are more distinctive. The apparent deviation of the material is reduced in 11 of the 13 materials. The reduction averages nearly 50% and for the extreme case reaches 75%. Statistically such a reduction would be expected as the probability of extreme values is greatly reduced by larger samples.
Summary

This study has developed an improved method of sampling. The data demonstrates a significant reduction in standard deviation of the samples. The method is easily applied to many aggregates and granular materials. The truck required is readily available in many areas of the country. In other areas, truck tailgates could be modified cheaply.

A philosophy of quality control for continuously varying bulk materials is established. The minimum quantity which will be expected to meet the gradation is a truck load. Rearrangement of particles from stage to stage of construction precludes the necessity for each possible small quantity meeting gradation requirements.

The use of truck size sampling will save time. The actual time for loading and handling will not differ greatly from current practices. However, the lost time, due to explaining misleading or erratic reports, will be largely eliminated as shown by the greatly reduced standard deviation of the samples. Of course, technical advantages related to more exact knowledge of materials characteristics will accrue to the mix designer and other interested parties. With time, more realistic specification limits can be established.

The voluminous numerical data for the 120 trucks and 825 samples processed for the second portion of this study are on IBM cards and can be made available where needed.
Sampling Procedure

Equipment for drawing a sample is not elaborate or even restrictive. The major item is a dump truck equipped with a restricted tailgate opening such as used for feeding a sand spreader. In northern states, such trucks are widely used for winter ice control, figure 7.

FIG. 7 TAILGATE USED FOR SAMPLING

The exact dimensions of the truck bed or even of the discharge opening are not important, other than insuring free flow of the material. A sampling box must be made for use with the truck. Figure 8. This is a wooden box with dimensions such that the entire flow from the tailgate opening can be caught. The box should hold 50 pounds of aggregate. That used in the study was 6 inches wide, 6 inches deep and 26 inches long with a rope handle on each end. The material was 3/4 inch plywood.
Aggregate can be sampled from any overhead bin or stock pile. The truck is loaded by gravity or loader. The body elevated and a flow through the door established. A rate of one ton per minute permits time for handling the samples. Once the flow is well established, the box is passed into the stream and the entire flow caught until the box is full. Figure 8. The box is immediately dumped and the process repeated at the mid point of the load and again near the end. Timing and handling are expedited if three boxes are used. The three parts thus obtained are mixed and quartered, giving a 75 pound sample which is logged and sent to the laboratory.
Detailed Analysis of Results

a. Samples at screen decks

Typical results from the samples taken at the screen decks are shown in figures 9 and 10. Figure 9 is for samples of 3/4" nominal size aggregate with the upper portion of the figure showing the percent passing a 3/4" sieve. The third portion of the figure shows the percent passing the 3/8" sieve. The solid line connecting small squares represents the data for a sample composed of equal portions from each of the four screen decks. The heavy line is the average of the test results for the four screen decks.

Figure 10 represents the test data from 3/8" nominal size stone and is the percent passing #4 sieve. For all samples, 100% passed the 1/2" sieve, 99 to 100% passed the 3/8" sieve, and less than 1% passed the #8 and #16 sieves. Consequently, the #4 sieve was the only significant one for this material. Looking at the curves of Figure 10, a significant change occurs in the percent passing the second screen deck between 11:30 and 12:15. The percent passing this screen nearly doubled in this period. There is a minor decrease in percent passing the third and fourth decks. It should be noticed that during this time interval both the average and the result of the compound sample from the four decks show only a minor change in percent passing. It is probable that, for some reason, the relative loading of the four decks varied at this time. The loading must have increased on the second deck and decreased on the third and fourth. This is of interest to the plant operator, as figure 5 also shows a change between 12:35 and 1:05 on the June 9th portion of the curves. A review of plant
FIG. 9  PERCENT PASSING VERSUS TIME
operations quickly revealed this change was due to a different loading of crushing and screening plant during the lunch break each day. After removing this variable, further curves resemble the June 24th portion of figure 10. This information is of interest to the plant operator, but as shown in the average curves, has little value to the consumer.

The average curve shown is the arithmetic average of the four individual screen curves. The composite curve is the result from samples made by combining equal weight of material from the four screen decks. The latter could be considered as a mechanical average. The accuracy with which either represents the gradation of the total product is affected by the assumption that equal weights are delivered from all screens. If either is to more nearly approach the correct value for the total, the relative weights contributed by each screen must be found. At this time, none of the plants visited are equipped to measure the weights delivered from individual screen decks. Statistical comparisons of the mechanical and arithmetical averages show no significant difference. Therefore, in all later work the two forms of average are used interchangeably.

b. Sampling Truck size

In order to get some indication of the importance of truck size on variation of sample obtained, figures 11, 12, 13 and 14 were prepared. Figure 11 shows data for 1-1/4" stone and presents standard deviation vertically against the weight of aggregate in the truck horizontally. One portion is the standard deviation of the material passing the 1" sieve and the other portion standard deviation of that passing the 1-1/4" sieve.
3/4" AGGREGATE
PASSING 1/2" SIEVE

Fig. 12 STANDARD DEVIATION VERSUS SIZE LOAD
Fig. 13  STANDARD DEVIATION VERSUS SIZE LOAD
Figure 12 is for 3/4" stone and shows the deviation of the material passing the 1/2" sieve vertically against the weight of material in the truck horizontally.

Figures 13 and 14 present in a similar manner data for 1/2" stone passing a 3/8" sieve and 3/8" stone passing a #4 sieve.

The first three figures show little correlation between weight of aggregate in the truck and the deviation within the samples taken. Computer fitted curves and correlation factors were not used as it is readily apparent that the deviations found did not correlate with size of load. Figure 10 implies some correlation from lower left to upper right. This trend is due to 2 of the 28 points and can not be considered as well established. Sampling intensity approximated one sample for each ton of material.

c. Effect of sample location on aggregate gradation

In a further attempt to determine the effect of size sample truck used, figures 15, 16, and 17 were prepared. These figures show the relation between percent passing a particular sieve and the time a sample was taken from the truck for 3 different size truck loads of 3/4" stone.

Figure 15 shows the percent passing the 3/4" sieve, the 1/2" sieve, and the 3/8" sieve for 3/4" stone for a truck load of 7,500 pounds. The 13 samples represent one sample per 500 pounds of stone. Figures 16 and 17 show similar plots for 14,000 and 24,000 pound truck loads with a sampling frequency of approximately one sample per ton. In each figure the average for the truck has been drawn and the standard deviation within the truck noted.
Fig. 15 VARIATION IN % PASSING PER TRUCK LOAD
Fig. 16 VARIATION IN % PASSING PER TRUCK LOAD
Fig. 17 VARIATION IN % PASSING PER TRUCK LOAD
Inspecting figures 15, 16, and 17 closely, one recognizes no significant trend with respect to percent passing as the material runs from the truck. The sample data has been plotted with the first sample from the truck on the left and the last sample on the right. In certain curves—that for the 3/8 on the 7,500 pound truck and the 14,000 pound truck and possibly for the 1/2 inch sieve on the 14,000 pound truck, the implication is that the material becomes finer, that is, a higher percent passes as the end of the load is approached. Yet, reviewing several hundreds of such curves, the reverse occurs about equally often.

4. Selection of number of samples

Considerable thought was given to the number of parts which should make up a sample. Basically the formula \( n = \frac{t^2 \sigma^2}{\delta^2} \) can be applied.

\( n \) = number of samples
\( t \) = the desired degree of assurance or probability of success in obtaining a correct answer, measured in standard deviation units from the center of the curve of distribution of \( t \)
\( \sigma \) = standard deviation of the measurement
\( \delta \) = the maximum allowable difference between the computed average of the measurements and the true average.

This formula is tedious to use as \( t \) is related to \( n \) and an iterative solution is required. If the standard deviation of the material is known, then \( n \) can be computed more directly. For this study, the many samples taken at the screen house were used to estimate the standard deviation of the material and establish \( t \) for the chosen level of confidence and thus determine \( n \). Table 1 presents the results of such computations.
Table #4  NUMBER OF SAMPLES REQUIRED FOR VARIOUS STANDARD DEVIATION, LEVELS OF CONFIDENCE, AND ACCURACIES.

<table>
<thead>
<tr>
<th>σ of material based on 50 samples</th>
<th>2σ=3</th>
<th>2σ=4</th>
<th>2σ=5</th>
<th>2σ=6</th>
<th>2σ=3</th>
<th>2σ=4</th>
<th>2σ=5</th>
<th>2σ=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.3</td>
<td>0.7</td>
<td>0.3</td>
<td>1.8</td>
<td>1.0</td>
<td>0.6</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.8</td>
<td>1.6</td>
<td>1.0</td>
<td>0.7</td>
<td>4.1</td>
<td>2.3</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>5.8</td>
<td>2.8</td>
<td>1.8</td>
<td>1.3</td>
<td>7.3</td>
<td>4.2</td>
<td>2.6</td>
<td>1.8</td>
</tr>
<tr>
<td>5</td>
<td>7.9</td>
<td>4.1</td>
<td>2.8</td>
<td>2.0</td>
<td>11.4</td>
<td>6.4</td>
<td>4.1</td>
<td>2.8</td>
</tr>
<tr>
<td>6</td>
<td>11.3</td>
<td>6.4</td>
<td>4.1</td>
<td>2.8</td>
<td>16.3</td>
<td>9.2</td>
<td>5.9</td>
<td>4.0</td>
</tr>
<tr>
<td>7</td>
<td>15.4</td>
<td>8.7</td>
<td>5.6</td>
<td>3.9</td>
<td>21.2</td>
<td>12.5</td>
<td>8.0</td>
<td>5.6</td>
</tr>
<tr>
<td>8</td>
<td>20.2</td>
<td>11.3</td>
<td>7.3</td>
<td>5.0</td>
<td>26.0</td>
<td>16.3</td>
<td>10.5</td>
<td>7.3</td>
</tr>
</tbody>
</table>

The diagonal lines represent n of 3. This appears to be an optimum number balancing effort against improvement. An example of the results would then be:

Table 1 shows, at the screen house, a standard deviation for the material passing the 1/2" sieve from 3/4" stone of 1.5%. Entering Table 1 with sigma of 1/2, at 90% confidence the three part combination sample should be within ±1% of the mean of the material. At 95% confidence the same sample would be within ±1.5% of the mean.

e. Combination samples

In place of further sampling, three part combination samples were produced synthetically by combining three sample results. For each truck, this was done by dividing the number of samples in that truck into three groups, then programming the computer to compute the averages for all combination samples of three parts, taken one from each of the groups. For example, truck load #1, source #1, of 1-1/4" stone, consisted of 6 samples. The combinations would then be samples 1-2-3, 1-3-4, 1-4-5, 1-4-6, 2-3-5, 2-3-6, 2-4-5, and 2-4-6 for a total of 9 combinations. Each combination was then considered as a single sample and the mean and
standard deviation of the 8 combination samples computed. A comparison of the means shows the mean of the combination is almost identical with the mean of the original samples. The minor difference in mean is explained by the way the combinations were taken. If the number of the samples in the truck was not divisible by 3, the computer program divided the number of samples by 3 and threw away any remainder.

The combinations averaged three individual gradations. If the three samples were mixed together and then graded, the same gradation would result. Mechanical versus computational averaging was discussed in Appendix 2, Section a.